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PLANE SURVEYING

A PRACTICAL TREATISE ON THE ART OF PLANE SURVEYING, INCLUDING CHAINING, LEVELING, COMPASS AND TRANSIT MEASUREMENTS, LAND AND CONSTRUCTION SURVEYING, TOPOGRAPHIC SURVEYING, AND MAPPING

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INTRODUCTION

SURVEYING is probably one of the oldest mathematical arts. In the past it was used only in parting off or dividing up land, but in the progressive twentieth century, it has become an essential requirement in all great engineering undertakings. The principles of surveying must be applied by the engineer in the preliminary survey; the transit and other surveying instruments are necessary for the bridge engineer to obtain an accurate location of right of way, piers, and abutments; and an accurate and practical working knowledge of surveying is absolutely required in irrigation work, river and harbor improvements, steel construction for office buildings, or in the alignment of tunnels.

❡ According to the old order of things the engineer, or surveyor, was a specialist—a dictator who made great engineering projects wait his convenience for the preliminary surveys before the actual work of construction could begin. Today every construction gang has its surveying engineer and in addition to this the superintendent has an accurate working knowledge of the use and adjustments of the level and transit so that he could do the work if necessary. This knowledge must be acquired through university training or from specially prepared handbooks, such as this, supplemented by practical work in the field. It is for those who must acquire their knowledge by their own individual efforts in home study that this text has been compiled.

❡ The utmost care has been used to bring the treatment of each subject within the range of common understanding. The language is simple and clear; the difficult technical terms and formulas of higher mathematics having been avoided and only

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the practical working formulas given. Thus a complete mastery of each subject can easily be obtained, since the treatment has been arranged to carry the reader on by easy and logical steps. This text appeals to the technically trained expert as a ready and easy review of Plane Surveying and gives to the man seeking practical training a thorough working knowledge of this interesting subject.

¶ The author of this work has had years of experience, not only in teaching the subject but in the practical side as well, and has given the reader a multitude of helpful suggestions for successfully carrying out the operations required. It is the hope of the publishers that this work will be found a worthy contribution to our standard technical literature.

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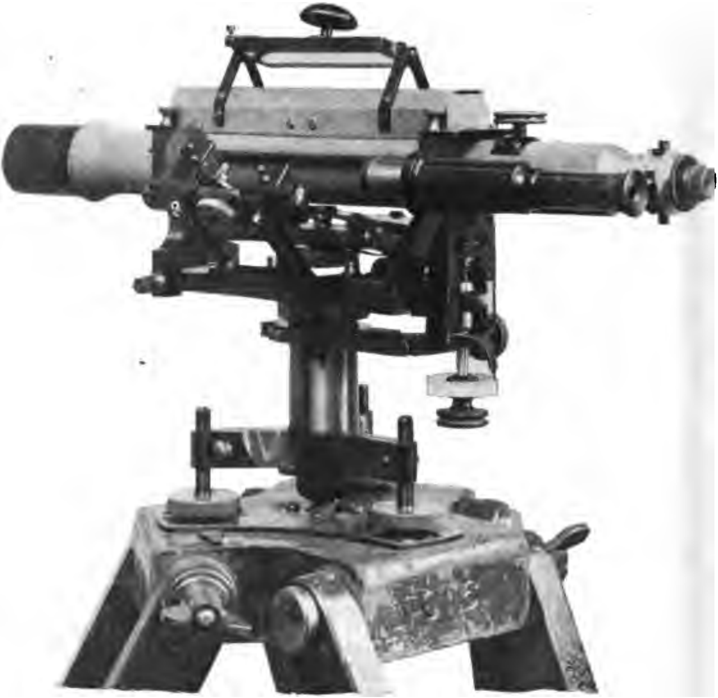


GEODESIC LEVEL USED BY COAST AND GEODETIC SURVEY

PLANE SURVEYING

PART I

INTRODUCTION

Definition of Surveying. Surveying is the art of making such measurements as are necessary to determine the relative positions of points or lines on the earth's surface and of keeping records of such measurements so that a drawing, called a *map* or *plat*, may be made; or, *vice versa*, it is the art of locating on the ground points shown on drawings.

Kinds of Surveying. There are two kinds of surveying, plane and geodetic. In *plane surveying*, the portion of the earth's surface included in the survey is regarded as a horizontal plane; in other words, the curvature of the earth's surface is neglected. In the ordinary operations of land surveying this assumption will not cause appreciable error as the lines and the areas dealt with are of a limited extent. As *geodetic surveying*, on the other hand, deals with extensive lines and vast areas, the effect of the curvature of the earth's surface must be taken into consideration.

Divisions of Plane Surveying. Plane surveys may be distinguished as: (1) *land surveys*, in which the object is to determine the boundaries of a section of land so they may be described in a deed, to locate the boundaries from a deed, or to compute the area; (2) *topographic surveys*, in which we are not particularly concerned with the boundaries but desire a map that will show all the topographic features, i.e., roads, hills, lakes, streams, etc.; and (3) *construction surveys*, in which we must locate on the ground, from a drawing, the stakes necessary for the contractor in constructing the bridge, the building, or whatever is shown on the drawing.

Length Measurement. *Horizontal Projection and Difference in Elevation.* Of the two kinds of surveying measurements—distance, or lengths, and angles—the distance measurements require final reduction to standard conditions. While it is not possible always to measure directly on the ground the distances which are used on the final map, all measurements are finally reduced to horizontal distances and vertical distances.

For example, in Fig. 1, the distance between the two stakes *A* and *B* measured along *AB* and known as the *inclined distance* (I.D.) is not given on the final map which shows instead the length *AC* or the *horizontal distance* (H.D.). In surveying, therefore, when we mention the distance from *A* to *B*, we always mean the horizontal distance.

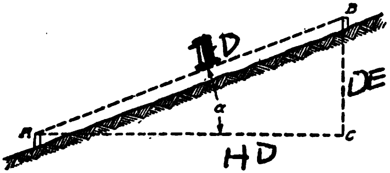


Fig. 1 Section of Survey, Showing I.D., H.D., and D.E.

In Fig. 1, *BC* is the vertical distance, or *difference in elevation* (D.E.).

Consequently, for reduction to standard conditions, if the inclined distance *AB* is measured and also the angle *a*, which is known as the vertical angle, then the horizontal distance *AC* equals *AB* times the cosine of the angle or, in general

$$\text{H.D.} = \text{I.D.} \times \cos a$$

Also the difference in elevation *BC* equals *AB* times the sine of the angle or, in general

$$\text{D.E.} = \text{I.D.} \times \text{sine } a$$

Projected Boundaries. It is required also, both by custom and by law, that areas of fields shall be computed, not from the actual length of the boundaries on the surface of the ground, but from their projection on a horizontal plane; hence, as used in surveying, the term *area of a field* means the area of the projection of the field on a horizontal plane and not the surface area.

Units of Length Measurement. The unit of length is the foot (ft.), which, however, is divided usually into decimals instead of into the usual inches and common fractions in order to facilitate computation. In common-fraction equivalents 0.01 foot equals nearly $\frac{1}{8}$ inch and 0.10 foot equals over $1\frac{3}{16}$ inches. In construction work feet and inches still are used. The engineer should learn the following equivalent values as they will be of general use in the field.

TABLE I

Gunter's, or Land, Measure

Linear	Area
1 link = 0.66 ft. = 7.92 in.	1 perch = 1 sq. rd. = 272¼ sq. ft.
1 rod = 25 links = 16½ ft.	1 rood = 40 perches = 10,890 sq. ft.
1 chain = 100 links = 4 rods = 66 ft.	1 acre = 4 roods = 10 sq. chains = 43,560 sq. ft.
1 mile = 80 chains = 5280 ft.	1 square mile = 640 acres = 1 section

⅛ in. = 0.0104 ft.
1 in. = 0.0833 ft.
2 in. = 0.1667 ft.
3 in. = 0.2500 ft.
4 in. = 0.3333 ft.
5 in. = 0.4167 ft.
6 in. = 0.5000 ft.
7 in. = 0.5833 ft.
8 in. = 0.6667 ft.
9 in. = 0.7500 ft.
10 in. = 0.8333 ft.
11 in. = 0.9167 ft.
12 in. = 1.0000 ft.

Gunter's Chain. Formerly, in land surveying, a special unit called the *chain*, Fig. 3, was used. It was devised by Gunter and equaled 66 feet, so that 10 square chains equaled 1 acre, that is, $10 \times (66)^2 = 43{,}560$ square feet, or 1 acre. The chain was divided into 100 links, and hence a *link* equals 0.66 foot, or 7.92 inches. This unit has been practically discarded and is now used only in public-land surveys and by a few surveyors who do mostly land surveying. Lengths in chains and links will be found on old land maps and deeds, also areas in roods and perches, and the equivalents given in Table I are useful in changing them to feet and acres.

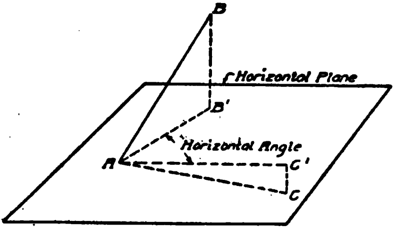


Fig. 2. Diagram Showing Method of Measuring Angles in Surveying

Angle Measurement. Angles always are measured directly in the field either as horizontal angles or as vertical angles and hence no reductions are necessary. Thus, in Fig. 2, when we measure the angle at A between the high point B and the low point C, we measure the angle $B'AC'$, or the horizontal angle between the projections of AB and AC on a horizontal plane through A. The vertical angle at A between C and B also is measured directly and equals the angle CAC' plus the angle $B'AB$.

Units of Angle Measurement. The units used in measuring angles are the *degree* (°), the *minute* ('), and the *second* ("), the full circle being divided into 360°. In some countries it is divided into 4 quadrants of 100° each, and it has been proposed also to use decimals of a degree instead of minutes and seconds.

Degree of Accuracy Practicable. In studying surveying, the student should remember that it is impossible to obtain absolutely accurate or true measurements and that to obtain a high degree of accuracy requires more time and care, and hence costs more than a less accurate measurement. While surveyors must be continually on their guard to detect and prevent errors in their measurements, they must also suit the accuracy of their work to the requirements. Thus, in making a survey of a piece of farm land worth $40 an acre, it would be a waste of time and money to do as accurate work as would be required in surveying a city lot worth more than that amount per square foot. Also, in making a survey, it would be a waste of time and money to measure the angles with a much higher degree of precision than was used for the distances.

Working Conditions. For any piece of work, therefore, a surveyor must know what degree of accuracy he should require; what methods to use to obtain this accuracy; and how to use the instruments and methods to obtain the required results.

EXAMPLES FOR PRACTICE

1. The inclined distance between two stakes is measured and found to be 220.32 feet. The difference in elevation of the tops of the stakes is 35.42 feet. (a) What is the vertical angle? (b) What is the distance between the stakes? Ans. (a) 9° 15' 05"; (b) 217.46 feet

2. Change the two given measurements in example 1 to feet, inches, and fractions, giving in each case the nearest $\frac{1}{8}$ inch.
Ans. (a) 220 feet 3$\frac{7}{8}$ inches; (b) 35 feet 5 inches

3. What is the length in feet of the side of a square containing 1 acre? Ans. 208.71 feet

4. The length of a boundary is given on an old map as 13 C 54$\frac{1}{4}$ L. What is the length in feet and decimals? Ans. 893.80 feet

CHAINING AND LEVELING

CHAINING, OR MEASUREMENT, OF DISTANCE

INSTRUMENTS

The instruments used in chaining consist of tapes; plumb bobs; range poles, or *flags;* and chaining, or marking, pins.

Pacing Method. Approximate measurement of distances may be made by pacing. An experienced man may measure in this way with an error of about 1 foot in 80 or 100 feet, i.e., in measuring a true distance of 80 feet, he will obtain values ranging from 79 to 81 feet.

Gunter Chain Superseded by Tape. The Gunter chain, Fig. 3, made of links of heavy wire was formerly used, as has been described, but it has been superseded by the tape because the chain was not only heavy and cumbersome but each link with its connections had six wearing surfaces, so that if each surface were worn away only 0.01 inch, the whole chain would elongate 6 inches. Again, the rings sometimes were bent out of true form or, in pulling the chain over the ground, they collected mud or weeds and the accuracy was very low. Tapes may now be obtained graduated in chains and links,

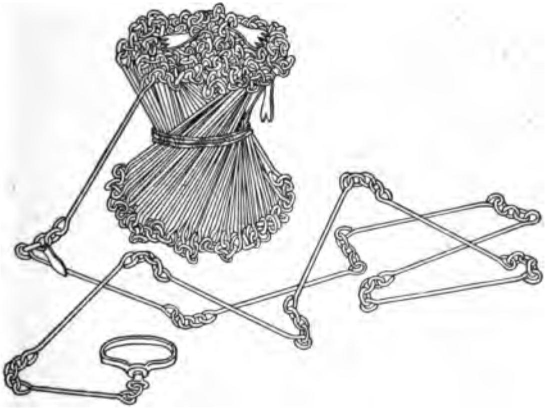


Fig. 3. Gunter Chain

and the process of taping is still called *chaining*, but the chain as an instrument for measuring distances is obsolete.

Tapes. Tapes are made of various materials and are, therefore, classed as linen, metallic, and steel tapes.

Linen Tapes. Linen tapes, from the nature of the material are likely to twist and tangle and, when wet, are easily stretched; for these reasons they do not long retain their standard length. They are used only in the roughest kind of work.

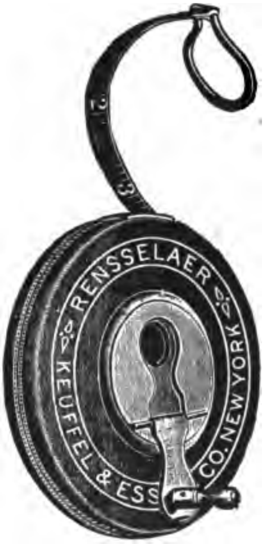


Fig. 4. Ribbon Tape
Courtesy of Keuffel & Esser Company, New York City

Metallic Tapes. Metallic tapes have a linen body with threads of copper or brass running throughout their length. These metallic threads prevent twisting and tangling and, in a general way, assist in preserving the standard length of the tape. They are better than linen tapes but are not suitable for accurate work.

Steel Tapes. Ribbon Type. There are two kinds of steel tapes—ribbon and band. Ribbon tapes, Fig. 4, are made of thin steel about $\frac{3}{8}$ inch wide, usually in lengths of 50 or 100 feet. (These dimensions cannot be made very specific as the great variety of tapes on the market make definite practice impossible.) They are divided into feet, 10ths of a foot, and 100ths of a foot, the divisions being etched upon the tape. Ribbon tapes are usually provided with a leather case for convenience when not in use, Fig. 4, and frequently have the inner end fastened in the case, but reels are used for band tapes and the tape may be detached from the reel, Fig. 5.

Ribbon tapes are used usually when considerable accuracy in measurement is required, as in laying out foundations for buildings and bridge piers or for measuring up sewer lines, etc. From the nature of their construction, they will not stand much wear and tear and therefore are not adapted to the rough usage of general field work. After use in the field, the tape should be carefully wiped off and oiled, if necessary, as the rust will obliterate the graduations and make them difficult to read. In using the ribbon tape in the field, care must be exercised to prevent its twisting and kinking or catching under sticks or stones, as a slight jerk will break it.

Steel-Band Type. Band tapes, Fig. 5, are best adapted to general field work and to rough usage. They are made of heavy steel $\frac{1}{8}$ to $\frac{5}{16}$ inch wide and 100 feet long, divided into feet, usually, the first and the last foot being divided into 10ths. The 1-foot divisions may be marked by rivets, although the rivets tend to weaken the tape, or the divisions may be sometimes marked by solder which is notched at the proper point and stamped with the number. The bands are fitted usually with light detachable handles for use in the field, but these are easily displaced or often lost in dragging the tape over stones or through grass; so it is better to fit the tape with a strip of rawhide belt lacing or leather about $\frac{1}{2}$ inch wide and 1 foot long. After use, the band tape should be gathered up in loops about 3 feet long and tied in the middle, forming a figure 8. If it is desired to wind the tape upon a reel, there are at present upon the market several styles of reels which are stiff in construction and convenient to carry.

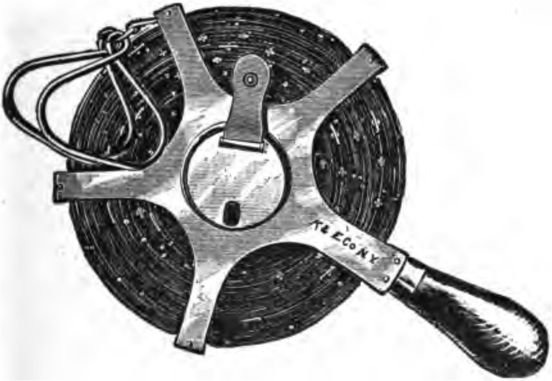


Fig. 5. Steel-Band Tape in Reel
Courtesy of Keuffel & Esser Company, New York City

Supplementary Chaining Devices. *Marking Pins.* In connection with the tape there should be provided a set of eleven marking pins from 15 to 18 inches in length, and to each pin should be attached a piece of red flannel to prevent its being overlooked in the grass. In practice, wood stakes, carefully numbered with crayon, are more generally used and may be found more convenient. However, for this simple chaining operation, the older practice will be described.

Flags. There should be two rods, called flags, or range poles, from 6 to 8 feet long, divided into foot lengths and painted alternately red and white. These rods are sometimes constructed of straight white pine, but a $\frac{3}{4}$-inch gas pipe fitted with a steel shoe is better.

Plumb Bobs. Plumb bobs also are required, a slender 12-ounce brass bob being desirable.

METHODS

Proper Conditions for Tape Measurement. *Required Tension.* When tapes are manufactured, the marks are put on the tape by using a standard length, the tape being supported throughout and a pull of 2 or 3 pounds applied to hold it straight. As the standard is correct usually for a temperature of 62° F., the tape will be of correct length at this temperature only and when supported throughout and under the standard pull. If the tape is used at any other temperature, its length will not be standard. In actual field work it seldom is supported throughout but sags between the ends and in order to keep it off the ground a greater pull is needed. In fact, the chainmen should remember that a harder pull on the tape is necessary in windy weather than when it is calm. In accurate taping, as described under Triangulation, "Plane Surveying", Part IV, these things must be allowed for and corrections applied In ordinary work, such as described above, the temperature is neglected, and as pull makes the tape longer and sag tends to shorten it, an attempt is made to make these counterbalance each other by applying a certain pull—about 10 or 12 pounds. A student should learn to estimate the required amount of pull by a little practice in pulling on a spring balance.

Comparison with Standard. The length of a tape is also liable to vary from time to time owing to excessive stretching and wear. For this reason, every surveyor should have a standard tape and should compare his field tapes with the standard. The standard tape may be simply an ordinary tape which is used for no other purpose. The length of the tape under standard conditions may be obtained at a slight cost by sending it to the U. S. Bureau of Standards, Washington, D. C.

Method of Correcting for Error in Tape. It sometimes happens that a distance is measured or laid out of an apparent length L' with a tape whose length l' is afterward found to be too long or too short by comparing it with a tape of standard length l. This may be corrected by the following method, in which the *true* distance or length of line is

$$L = \frac{l'}{l} \times L'$$

For example, suppose a line as measured is found to be 625 feet in length, and the 100-foot tape used in this measurement is afterward found to be too long by 6 inches. Then, to find the correct length, we substitute the proper values in this equation, obtaining

$$L = \frac{100.5}{100} \times 625 = 628.125 \text{ feet}$$

The true length of the line, therefore, is 628⅛ feet.

This method of correcting results for errors in the tape can conveniently be put in axiomatic form, thus

When the tape is *short* the line is too *long*
When the tape is *long* the line is too *short*

General Precautions in Using Tape. The following points are important in using a tape:

(1) Examine the tape and make sure just where the zero mark is. Tapes of the type shown in Fig. 4 sometimes are made with the zero at the end of the tape, at the inner side of the ring, or at the outside of the ring. Bend the end of the tape over, putting the 1-foot mark even with the 4-foot mark and noting where the 5-foot mark comes.

(2) Be sure the tape has no twists or kinks in it and that it is not deflected to one side by grass, sticks, or stones.

(3) Never straighten the tape with a jerk; raise it clear of the ground, straighten and stretch it with a steady pull, and lower it steadily into place.

(4) In taping, the tapeman should never brace himself against a pin.

(5) In all tape measurements, every reasonable effort should be made to hold the tape level.

Ordinary Chaining on Level Ground. For measuring a line with the 100-foot tape, two men are required, a *head* and a *rear* chainman. The first step is to set one of the flags at the far end of the line to be measured or, if the line is too long, only as far ahead as can be seen distinctly. It is best to mark the beginning of a line with a stake (called a *hub*) driven as closely down to the ground as circumstances will permit. The tape is then thrown out in the direction of the line, the 100-foot mark going ahead. The head chainman takes the pins and the forward end of the tape and with a flag walks off in the direction of the forward end of the line dragging the tape after him. When nearly 100 feet away, the rear chainman cries "chain", and the head chainman faces back and holds the flag vertically to be signaled into line by the rear chainman. The tape is then straightened and stretched taut and a pin is stuck into the ground at the 100-foot mark. The head chainman then picks up his end of the tape and starts off as before, the process being repeated each time the tape is moved; except that the rear chainman must be particular to pick up each pin that is left in the ground by the head chainman.

If the line is more than eleven tapes in length, after the head chainman has stuck his last pin, he cries "pins", and the rear chainman delivers to him the ten pins that he has picked up. If the line to be measured is very long, some method should be adopted for *keeping count of the number of times the pins have been exchanged.* If the line ends with less than the length of a tape, the head chainman pulls out the tape to its full length—not sticking a pin, however—and then walks back and notes the distance from the last pin to the end of the line. This distance added to the number of pins held by the rear chainman, including the last one stuck, will give the distance from the point at which the pins were exchanged. For instance, if the rear chainman has six pins and the end of the line is 65 feet from the last pin, the entire distance from the point of exchange of pins is 665 feet. It must be remembered that each exchange of pins counts for 10 tape lengths, or 1000 feet.

Chaining on Slopes. One of the most important uses of the tape is to measure distances accurately where the surface of the land is uneven or of a sloping nature, Fig. 6. In measuring up or down a slope, one end of the tape is raised until both ends are practically at the same level. If the slope is too steep to permit one end of a full tape being raised enough to bring the tape horizontal, the tape is *broken*, that is, only a part of the tape is used at each measurement. To do this, the tape should be stretched to its full length, the head chainman returning to such a point upon the tape that the portion between himself and the rear chainman may be properly leveled. A measurement is made with this portion; then the operation is repeated with the next section of tape, and so on until the entire tape has been used. Care should be taken not to confuse the pins. The high end of the tape may be transferred in any one of several ways, depending upon the degree of accuracy required. For great accuracy, a plumb bob should be used, but it should not be dropped, and the pin should not be placed in the hole made; instead, it should be placed about where the bob will drop, the grass should be tramped down, and the ground smoothed. The bob should be lowered carefully until it almost touches the ground and then allowed to come to rest. It should then be lowered until it reaches the ground, when the pin should be stuck in the ground slantwise across the line exactly at the point of the bob. If less accuracy is permissible, it may be sufficient to drop the pin, ring end down, and note where it strikes the ground, or a pebble may be dropped in the same way. In measuring uphill, the rear chainman must hold the bob directly over the pin in the ground while he aligns the head chainman and sees that he sticks

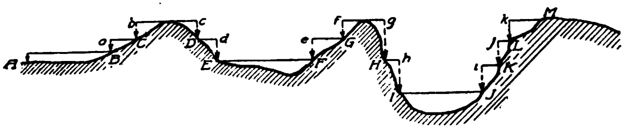


Fig. 6. Diagrammatic Section Showing Method of Chaining on Slopes

the pin when the bob is directly over the point in the ground. It is much easier to measure downhill than uphill, so that when close measurements are required on slopes, the measurement should, if possible, be made downhill. Even under the most favorable conditions, measuring lines with a tape is a most difficult operation for experts, and beginners cannot expect to attain proficiency except by constant practice and careful attention to every detail that will tend to eliminate error.

ILLUSTRATIVE EXAMPLE

Let it be required to find the distance from *A* to *M*, Fig. 6, at which points two hubs have been established. Let *A*, *B*, *C*, *D*, *E*, *F*, etc., be the points of the successive chaining and *a*, *b*, *c*, *d*, etc., the horizontal planes. Starting from the point *A*, suppose that the surface between *A* and *B* has no great differences in elevation and that the full length of the chain can be used between these two points. The head chainman goes to the point *B* and holds the head end of the chain on the ground, while the rear chainman holds the zero end at *A* and plumbs down with aid of the plumb bob, thus chaining the distance horizontally. This distance measured, the head chainman establishes a pin in the ground and calls out the distance "one hundred", or "station one", then goes to *C*, which must be approximately low enough to allow the rear chainman conveniently to plumb down to *B*. In this case the slope of the hill is greater than that between *A* and *B*, and thus the impracticability of using the full length of the chain is apparent. This distance, therefore, is taken at a fractional part of the chain, this, as before stated, being the process of breaking the chain. The distances at such breaks should, whenever possible, be taken at an even number of feet and at distances that are easily remembered, as 10, 20, 25, 50 feet, etc. The head chainman, in every instance, calls out the distance of such breaks and the rear chainman goes to the next peg and holds off the number of feet previously called out. Now, as the distance *AB* is 100 feet and the distance *BC* is 40 feet, the head chainman at *C* calls out "1 plus 40", meaning 140 feet from *A*. He next goes to *D*; then the rear chainman calls out the distance measured "1 plus 40", holds off 40 feet at *b*, and plumbs down to *C*. In this case the head chainman also plumbs down from *c* to *D*. This method is continued until *M* is reached, using the system of 1, 2, 3, 4, etc., plus the fractional measured distance, instead of using the whole number, as 125, 225, etc.

The rear chainman should gather the pins after a new point has been established. As already stated, the chaining can be checked by counting the pins picked up. Always allow the last pin to remain in the ground until absolutely certain that it is no longer desired and can be of no further service. It is important that the distances should be checked by both chainmen, as it may prevent serious mistakes and, in some cases, prevent rechaining the entire distance.

Measuring with Tape Having Only Decimal Ends. To measure a distance to 100ths, using a tape on which only the end feet are graduated to 10ths, proceed as follows: Measure the distance to the nearest foot (say it is between 51 and 52 feet); let the head chainman hold the 52-foot mark on the front point; it will then be 51 feet back to the 1-foot mark, and the rear chainman may read the 10ths and estimate the 100ths on the first foot of the tape, which is graduated.

Correction of Errors in Chaining. *Compensating Errors and Cumulative Errors.* Of the errors that occur in chaining, classed as either compensating or cumulative, the *compensating errors* are those which are liable to occur in either direction and hence tend to compensate. They are not so important as *cumulative errors,* which always occur in the same direction and hence tend to add up, or accumulate, and to seriously affect the accuracy of the work. In ordinary chaining the principal cumulative errors are those due to not keeping the tape horizontal and to not keeping it on line. Both have the same effect as using a tape that is too short; hence most measurements of distance result in values that are too big. They should be guarded against, and a beginner in chaining should use a hand level, Fig. 52, to test when the tape is horizontal, until he becomes expert enough to estimate this.

The accuracy of ordinary chaining is usually from 1 in 1000 to 1 in 2000 or more, depending on the roughness of the country over which the line is chained, etc.

Examples. 1. A line as measured with a certain 100-foot tape is 580 feet in length. It is found afterward that the tape is 0.3 foot too short. Determine the true length of the line. Ans. 578.26 feet

2. Two men are taping up a hillside; one is an experienced man and the other is not. (a) Should the better man be head or rear tapeman? (b) In going downhill, which position should he occupy? Ans. (a) rear; (b) head

3. A length of 420 feet was laid off on the ground with a tape afterward found to be 0.10 foot too short. What correction should be made in the distance as laid off? Ans. It should be increased by 0.42 foot

Errors in Area Due to Incorrect Tape. When an area has been measured with a certain tape that is afterward found to be in error, the corrected area may be found by the following rule: *The square of the true length of the tape $(l')^2$ is to the square of the length of the standard tape l^2 as the true area A is to the measured area A'; or the true area is*

$$A=\frac{(l')^2}{l^2}\times A'$$

Example. A certain field measured with a Gunter's chain was found to contain 625 acres, and the chain afterward was ascertained to be ¼ foot too long. What is the true area of the field? Ans. 629.74 acres

ANALYSIS OF PROBLEMS IN CHAINING

Linear Measurements. We will now analyze some of the problems which come up in the use of the chain or the tape in the field.

Perpendicular at Given Point on Line. Let AB, Fig. 7, be the given line and C the point in the line at which it is desired to erect a perpendicular. Since a triangle having sides in the ratio of 3, 4, and 5, or any multiples of these, will contain a right triangle, take parts of the chain or tape representing these distances, or multiples, and have the angle included between the shorter sides located at C. Fasten one end of the tape at E, 30 feet from C, and the 90th foot at C. Then, with the 50-foot mark in one hand, walk away from BC until both of the segments DE and DC are taut. Stick a pin or a stake at D, and DC will be the perpendicular required.

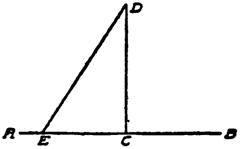


Fig. 7. Diagram Showing Method of Erecting Perpendicular at Given Point on Line

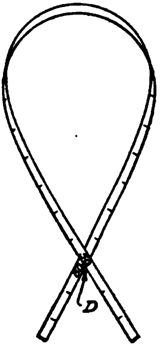


Fig. 8. Looping Tape to Obtain Measurements Required for Fig. 7

A better determination of point D can be made if the 100-foot mark is held at C and the tape crossed in a loop with the 50- and 60-foot marks opposite each other at D, as shown in Fig. 8. This gives a sharp corner at D instead of a circular bend.

If the perpendicular should be longer than can be laid out with the tape or chain, lay out CD as described and align a flag from C to D produced.

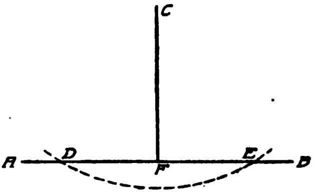


Fig. 9. Diagram for Drawing Perpendicular to Given Line from Given Point Outside

Perpendicular to Given Line from Given Point Outside. Let AB, Fig. 9, be a given line and C a point. From C as a center, with any convenient length of tape as a radius, describe the arc DE, cutting the given line at points D and E. Stick pins at D and E and, measuring the distance, bisect it at F. Then CF will be the perpendicular required.

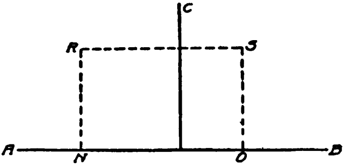


Fig. 10. Diagram for Fig. 9 when Given Point Is Distant from Given Line

If the line AB is too far from C to be reached with the tape, it will be necessary to range out a line conveniently near to C which shall be parallel to AB. To do this, erect a perpendicular at any convenient point on AB, as at N, Fig. 10, and prolong it as far as necessary, as R. Also, at a similarly located point O erect a perpendicular OS of the same length as NR. Then RS is parallel to AB, and the perpendicular let fall from C upon RS and prolonged to AB will be perpendicular to AB.

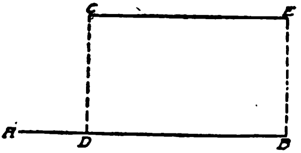


Fig. 11. Diagram for Drawing Line through Given Point Parallel to Given Line

Line through Given Point Parallel to Given Line. The given point and the given line being accessible, let C, Fig. 11, be the given point and AB the given line. From point C let fall and measure CD perpendicular to AB. At B erect a perpendicular BE equal to CD. Then CE is parallel to AB.

Line Prolonged beyond Obstacle. Let AB, Fig. 12, be the given line which is intercepted by a tree, a house, or other obstacle. It is required to locate the line CD, which will be in the direction of AB produced. At B erect BE perpendicular to AB and of sufficient length to clear the obstacle, and at F erect a perpendicular FG equal to BE. Prolong GE, erecting perpendiculars at H and I equal to BE, and obtaining points C and J which locate CD. Note that in order to make the error small, the offsets BE, FG, etc., should be small and far apart. The distance from A to D equals AB plus EH plus CD.

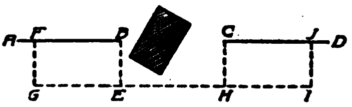


Fig. 12. Method of Prolonging Line beyond Obstacles

Line not Directly Measurable on Account of Obstacles but with Both Ends Accessible for Sight. At each end of the line erect perpendiculars of equal length sufficient to clear the obstacles and measure the length of the line between the extremities of these perpendiculars.

Alignment of Points on Line, Both Ends Being Accessible but Invisible from Each Other. A case of this kind occurs when it is desired to run a line across a wooded field, the trees and underbrush preventing the alignment of the intermediate stations. Let AB, Fig. 13, be the line whose length is desired. From A run a line AB', called a *random line*, in any convenient direction and continue it, until the point B can be seen from B'. At B' erect and measure $B'B$ perpendicular to AB' If any other length AC' is measured along AB', then $C'C$ perpendicular to AB' equals $B'B$ times AC' divided by AB', thus locating C on the line AB. Other intermediate points may be found in the same way, the line cleared, and the distance measured.

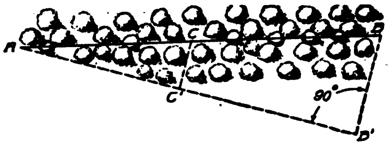


Fig. 13. Diagram for Locating Points on Line, Both Ends Being Accessible but Invisible from Each Other

The length may be found also by computation, knowing AB' and $B'B$.

Points in Line over Hill with Both Ends Visible from Summit. Set a flag at each of the points A and B, Fig. 14. As shown in the plan view, one man goes to D, as closely in line with A and B as can be estimated, and he then signals a man at *C* into line with *A*. *C* then signals *D* into line with *B* at *D'*. *D'* signals *C* into line with *A* at *C'* and so on, alternately, until the points *C''* and *D''* are reached in line with *A* and *B*.

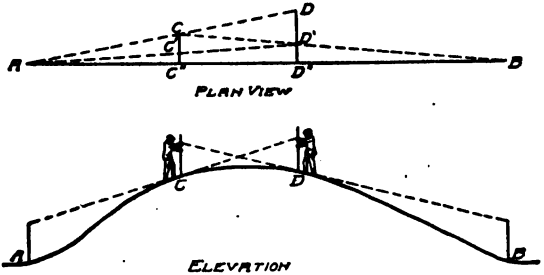


Fig. 14. Diagram Showing Method of Running Line over Hill with Both Ends Visible from Summit

If the points *A* and *B* cannot be seen from the top of the hill, run a random line over the hill, as described in the preceding paragraph, and offset it to the true line.

Points in Line across Wide Deep Valley. The extremities of the line being visible from each other, fix a point *C*, Fig. 15, on the edge of the slope in line with *A* and *B*. Then, holding a plumb line at *C* and sighting across to *B*, without changing position, glance down the plumb-bob string and line in *F*, etc.

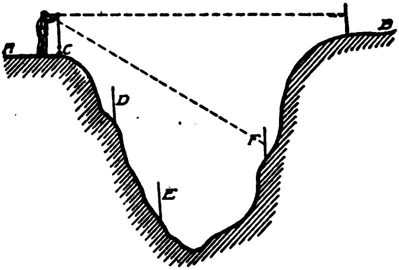


Fig. 15. Method of Locating Points on Line over Wide Deep Valley

Angular Measurements. *To Lay Off a Given Angle with a Tape* To lay off an angle with a tape, the procedure is as indicated in Fig. 16, where, from a point *A* on the line *BA*, it is desired to lay off a line *AD* that shall make an angle a of, say, 20° 10′ with *AB*. By trigonometry, if *AD* and *AC* are the equal sides of an isosceles triangle whose length l is 50 feet each, the size of the base b of such a triangle may be found from the equation

$$b = 2l \sin \frac{a}{2}$$

and accordingly, *DC* equals 100 times the sine of 10° 5′, or 17.51 feet (see Table of Natural Sines in Trigonometry). Hence, locate *C* on line *AB* 50 feet from *A* and fasten one end of the tape there. Holding the 17.51-foot mark even with, say, the 25-foot mark, the 75-foot mark being held at *A*; walk out toward *D* until the lengths of the tape *CD* and *DA* become taut; *D* then fixes the line *AD* at the desired angle with *AB*.

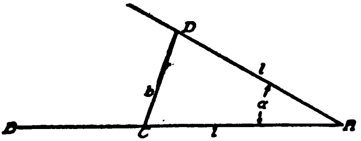


Fig. 16. Diagram for Laying Off Given Angle with Tape

To Measure Size of an Unknown Angle. The size of an unknown angle may be measured in the same way, in which case the sine of $\frac{a}{2}$ equals the base b divided by $2l$. For example, in a case like that shown in Fig. 16, let the two lines *AD* and *AB* be given and the angle *DAB* be required. Suppose that a length of 50 feet is laid off on *AD* and *AB* and that *CD* is measured and found to equal 27.98 feet. Then, from the relation $\sin \frac{a}{2} = \frac{b}{2l}$, the included angle a is found to be 32° 30′.

Measurement of Areas. Surveys usually are made by using a tape to measure the lengths and a transit to measure the angles. A complete survey of an area, however, may be made with a tape alone by two methods, namely, the triangular field method and the quadrangular field method.

Triangular Field Method. Let us consider the triangular field *ABC*, Fig. 17. Beginning at any convenient corner, as *A*, measure from A to B, then from B to C, and finally from C to the point of beginning. Should a stream cut across the field as shown, measurements should be made from the corners to the points where the stream crosses the boundary lines. Should it be found impossible to measure the sides of the field directly, owing to zigzag fences or other obstacles, offset the parallel lines, as shown in Fig. 17, and measure the length between such parallels.

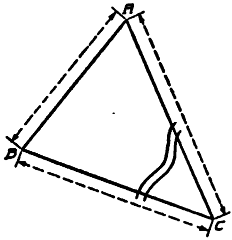


Fig. 17. Triangular Field Method of Measuring Areas

The area A of a triangle, in which the perimeter is $2S=a+b+c$, may be found by the following rule: *From one-half the sum of the three sides S subtract each side a, b, and c separately; multiply together the half sum and the three remainders and extract the square root of the product.* Or, the *triangular area* is

$$A=\sqrt{S(S-a)(S-b)(S-c)}$$

This rule is illustrated in section 68 of Trigonometry.

If the lengths of the sides are given in chains, the area will be given in square chains; if the lengths of the sides are given in feet, the result will be in square feet.

Quadrangular Field Measurement. To survey a four-sided field with the tape, measure around the field in the same way as before, also measure a tie line between two opposite corners, thus dividing the figure into two triangles, the sum of whose areas will give the area of the entire figure. Such a tie line is shown in Fig. 18 by the dotted line CA.

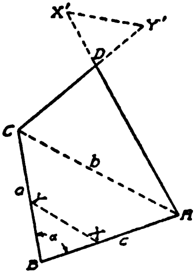


Fig. 18. Quadrangular Field Method of Measuring Areas

If neither of the diagonals CA nor BD can be measured conveniently, measure the short tie line XY, for instance, and the equal distances BX and BY. Then, in the isosceles triangle YBX three sides are given from which to find the angle YBX, or a. Having this angle, we can calculate the length of CA and therefore the area of the two triangles composing the field. Unless the triangles are isosceles so that the equation

$$b=(a+c)\sin\frac{a}{2}$$

may be employed for ascertaining the distance b, as just described in connection with angular measurement, the diagonal may be found by using the general relation for the third side of a triangle when two sides and their included angle are known, as explained in sections **56** and **57** of Trigonometry. In the former case the distance b would be found from the equation

$$b=\sqrt{a^2+c^2-2ac\cos a}$$

If it is not convenient to measure the tie lines inside the field, the two adjacent sides AD and CD may be prolonged to X' and Y', forming the tie line $X'Y'$. It will be found more convenient usually to lay off DX' equal to DY', thus forming an isosceles triangle.

Examples. 1. Given the three sides of a field as 5.25, 6.50, and 4.60 chains. Find the area of the field in acres and square rods.

Ans. 1 acre 31.46 square rods

2. In Fig. 18: Given BA equals 240.9 feet; BC equals 188.1 feet; BX equals 33.0 feet; and YX equals 42.9 feet. Calculate the area of the triangle BAC.

Ans. 0.514 acre

Use of Offsets. *Field Bounded by Stream.* To find the area of a field which is bounded in part by a stream, it is necessary to use offsets, as follows: Measure the regular sides of the field in the usual manner, and for the irregular boundary run a straight line, as ED, Fig. 19, and calculate the area of the field included between these straight boundary lines. To this area must be added the area included between the line ED and the irregular boundary. To find this area, erect perpendiculars from points along ED to the irregular shore line at such distances that the shore lines *1′–2′*, *2′–3′*, etc., may be considered straight. The desired area will evidently equal the sum of the areas of the trapezoids thus formed. The distance from E to any point *1*, *2*, or *3* on ED is called the *abscissa* of that point, and the perpendicular distances from ED to *1′*, *2′*, *3′*, etc., are called the *ordinates* of the point.

Let the ordinate at E be a and let the ordinates at *1–1′*, *2–2′*, *3–3′*, etc., to *7–7′*, be represented, respectively, by b, c, d, etc., to h;

also let the abscissas *E–1*, *1–2*, *2–3*, etc., to *6–7*, be represented, respectively, by a', b', c', etc., to h'.

$$\text{area of 1st trapezoid} = \frac{a+b}{2} \times a'$$

$$\text{area of 2d trapezoid} = \frac{b+c}{2} \times b'$$

$$\text{area of 7th trapezoid} = \frac{g+h}{2} \times h'$$

The total area will be the summation of these small trapezoids *1+2+*........*7* and will be equivalent to the following:

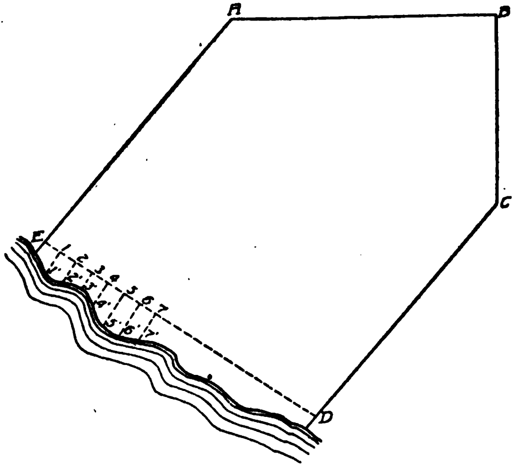


Fig. 19. Measuring Irregular Areas by Method of Offsets

$$2A = (a-c)a' + (b-d)(a'+b') + \text{, etc., } \ldots\ldots (g+h)(a'+b'+\ldots\ldots h')$$

It should be noted that this illustration gives the area from E to *7* only.

Instead of summing the trapezoid as above, the desired area may be found by the following rule: *Multiply the difference between each ordinate and the second succeeding one of the abscissa of the intervening ordinate, and multiply the sum of the last two ordinates by the last abscissa; then one-half the algebraic sum of these several products will be the area required.*

Inaccessible Area. To find the area of an inaccessible swamp, a lake, or other area, run a series of straight lines enclosing entirely the given area; and since the diagonals cannot be measured, measure tie lines either inside or outside of the area. Calculate the area included between the straight boundary lines, as already explained, and from this area subtract the area included between offsets let fall from points upon these boundary lines.

ILLUSTRATIVE EXAMPLES

1. Given the distances measured along the straight line *AB*, Fig. 20, with the corresponding offsets measured to the broken line *ACDE*; it is required to compute the area between *AB* and the broken line *ACDE*. Following the rule for trapezoidal areas, the data may be tabulated as follows:

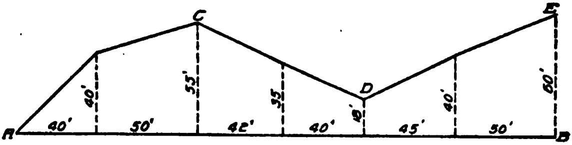


Fig. 20. Diagram of Irregular Boundary with Positive Offsets Only

Ordinates	Difference (ft.)	Abscissas (ft.)	Product (sq. ft.)
1 and 3 2	$0-55=-55$	40	$40\times(-55)=-2{,}200$
2 and 4 3	$40-35=5$	$40+50=90$	$90\times5=+450$
3 and 5 4	$55-18=37$	$90+42=132$	$132\times37=+4{,}884$
4 and 6 5	$35-40=-5$	$132+40=172$	$172\times(-5)=-860$
5 and 7 6	$18-60=-42$	$172+45=217$	$217\times(-42)=-9{,}114$
Last two Last	$40+60=100$	$217+50=267$	$267\times100=+26{,}700$
			32,034
			−12,174
			+19,860

One-half the algebraic sum of the products as tabulated above gives the required area, or 9930 square feet.

2. Given the distances measured along the straight line *AB*, Fig. 21, with the corresponding offsets measured to the broken line *ACDEFB*. Find the area between *AB* and *ACDEFB*. Check the

result by calculating the areas of the trapezoids and triangles of Fig. 21. Ans. 11,875 square feet

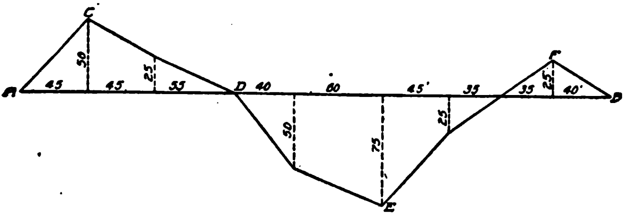
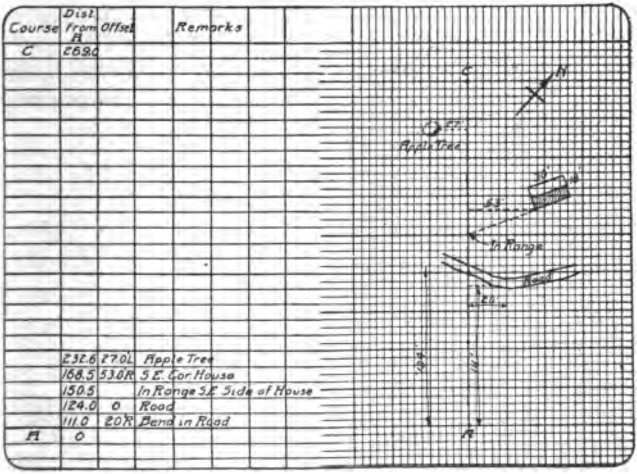


Fig. 21. Diagram of Irregular Boundary with Positive and Negative Offsets

Field Notes. *Clearness and Fullness Requisite.* In keeping field notes, clearness and fullness should be kept constantly in mind. As field notes often pass into other hands the notes should admit of but one interpretation to a person at all acquainted with the nature of the work. Extra time spent in the field in acquiring data may avoid confusion and vexatious delays when the notes are worked



Course	Dist from A	Offset	Remarks
C	269.0		
	232.6	27.0L	Apple Tree
	168.5	53.0R	S.E. Cor House
	150.5		In Range S.E. Side of House
	124.0	0	Road
	111.0	20R	Bend in Road
A	0		

C
N
27'
Apple Tree
30'
40'
53'
In Range
Road
20'
A

Fig. 22. Typical Notebook Page Showing Notes for Left Boundary of Survey Shown in Fig. 23

up in the office. Avoid the habit of keeping notes upon scraps of paper or in vest-pocket notebooks. Provide notebooks especially adapted to the keeping of field records and number and index them so that the contents may be understood at a glance. Remember that sketches made upon the ground aid materially in interpreting field notes that otherwise might be unintelligible.

Methods of Recording Notes. There are three principal methods of keeping field notes: by sketches alone; by notes alone; and by full notes supplemented by sketches.

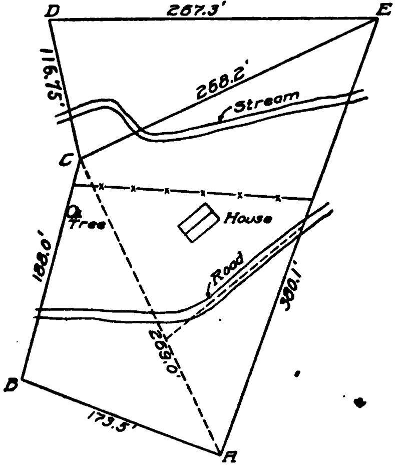


Fig. 23. Survey Map Made Up from Notes Shown in Fig. 22

The third method is, without doubt, the best. Fig. 22 shows the notes for one side or course of the survey shown in Fig. 23. These notes are made in the field, with a hard pencil, in one of the standard field books that are used for this purpose—shown also in "Plane Surveying", Part IV—and read from the bottom of the page up, as is usual, the notes being on the left-hand page and the sketch on the right-hand page. Note the method of locating interior objects which it is desirable to show on the final map by means of offsets and by getting in range. Some dimensions to illustrate this point are shown in Fig. 22.

LEVELING OR MEASURING DIFFERENCES IN ELEVATION

USE OF VERNIER

Principles of Vernier Operation. The *vernier* is an auxiliary scale for measuring, with greater precision, the spaces into which the principal scale is divided. The smallest reading of the vernier, or the *least count,* is the difference in length between one division on the main scale and one on the vernier.

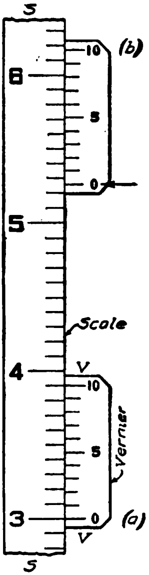


Fig. 24. Linear Vernier and Scale

Direct Vernier. The construction and the use of the vernier are shown in Fig. 24. Assuming the *main scale SS* to be divided into, say, 10ths of a foot, which are represented by the numbers, the smaller divisions will be 100ths, and a *vernier VV* that will enable us to read the scale to 1000ths is made as shown in Fig. 24(*a*). The space equal to 9 divisions on the scale, or 0.09 foot, is divided into 10 parts on the vernier. Thus each part on the vernier is 0.009 foot long or is less than the smallest division on the scale by 0.001 foot, which is the least count of this vernier.

If the vernier is moved to any position, as shown in Fig. 24(*b*), and we desire the closest reading of the distance of the zero of the vernier—marked by the arrow—above the bottom of the scale, we *first read the scale, then add to this the reading of the vernier.* Thus the scale reads 0.52, and following up the vernier until a line that coincides with a line on the scale is found, we get 6, or the vernier reads 0.006, and the final reading is 0.526. The reason for this is apparent at once if it is noted that the 5 mark on the vernier is 0.001 above the 0.57 mark on the scale and the 4 mark is 0.002 above the 0.56 mark, etc. This vernier is the type used on leveling rods, Fig. 32, and is called a *direct vernier.*

Retrograde Vernier. A retrograde vernier would be made by dividing 11 divisions on the scale into 10 on the vernier, but this type is seldom used.

Amount of Least Count. It may be easily proved that the least count of a vernier equals the value of the smallest division s on the scale divided by the number of divisions n on the vernier, or *least count* $= \frac{s}{n}$. A vernier as applied to a graduated arc is shown in Fig. 25. It will be noticed that the main scale is divided so as to read directly to 30′, 29 of these parts being divided into 30 vernier spaces; therefore the least count of this vernier is 1′.

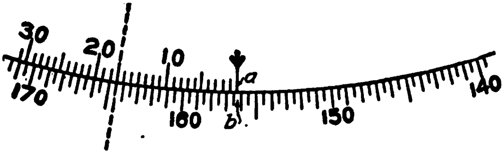


Fig. 25. Angular Vernier and Scale

Method of Reading Vernier. Readings are taken in the direction of the increasing graduations of the main scale. Thus, for example, in Fig. 25, it will be noted that the zero a has passed the 156th degree on the main scale and is near the 30′ or half-degree division at b. Therefore the coinciding lines of the vernier and the main scale must be between 0 and 30′, and we find them, by looking along the scale of the vernier, at 17′ Hence the reading is 156° 00′+17′, or 156° 17′.

Engineer's Transit Vernier. The usual form of vernier for an engineer's transit is shown in Fig. 26. The graduations are marked so that the arc may be read in either direction, and two verniers extending from the same zero to the right and to the left are used. The right-hand vernier and the inner row of numbers which slant to

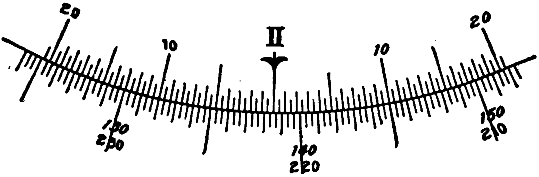


Fig. 26. Engineer's Transit Vernier and Scale

the right are used together, and for the other vernier the outside numbers are used.

It will be noticed that the main scale reads directly to 20′ and that the vernier is divided into 40 spaces. The least count of the vernier is therefore $\frac{1}{40}$ of 20′, or $\frac{20}{40}$ of 1′, which equals 30″.

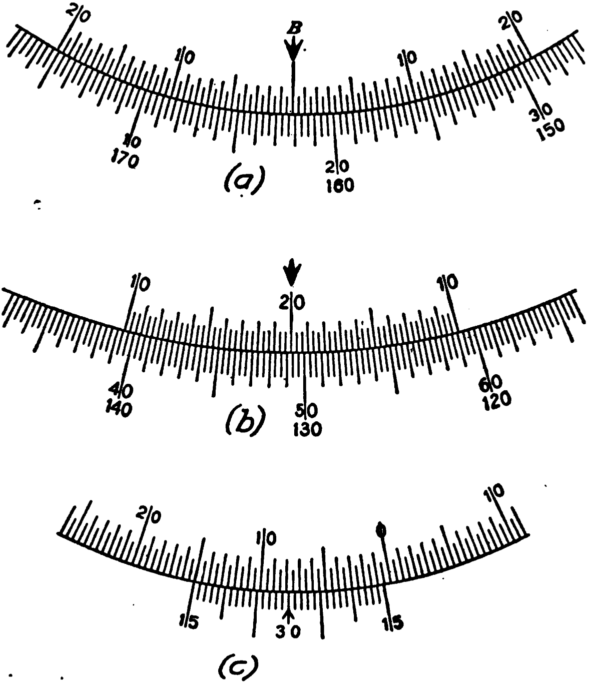


Fig. 27. Various Types of Angular Verniers

To read the inside scale, it will be noticed that the zero of the vernier is beyond the 138° mark and about halfway between the first and the second 20′ divisions. The reading so far, then, is 138° 20′. Now look along the vernier to the right until a line upon the vernier is found that seems to be a prolongation of a line upon the scale. This occurs at the division marked 10 upon the vernier, so that the reading is 138°·20′+10′, or 138° 30′.

For the outside scale, the zero of the vernier is beyond the 221° mark and about halfway between the first and the second 20′ divisions. The reading so far is therefore 221° 20′. Now look along the vernier to the left, as before, and the divisions coincide at the division marked 10 upon the vernier, so that the reading of the outside scale is 221° 20′+10′, or 221° 30′. The sum of the readings of the two scales equals 360°, as it should.

EXAMPLES FOR PRACTICE

1. Determine the least count and give the reading for the vernier in Fig. 27(*a*), 39 spaces upon the scale being equal to 40 spaces upon the vernier.

2. Determine the least count and give the reading for the vernier in Fig. 27(*b*), 59 spaces upon the scale being equal to 60 spaces upon the vernier. In Fig. 27(*b*) is shown what is called a *folding vernier*. To read it, follow along the vernier in the usual way until the division marked 10 is reached. If there are no corresponding lines, then go back to the other end of the vernier, begin with the other 10 mark and follow it back toward the center of the vernier.

3. Determine the least count and give the reading for the vernier in Fig. 27(*c*). This figure represents the usual method of dividing the vertical circle of the transit.

USE OF SPIRIT LEVEL

Level Bubble. The spirit level is a glass tube nearly filled with a mixture of ether and alcohol, the remaining space being occupied with the vapor of ether. If the tube were perfectly cylindrical, Fig. 28(*a*), the bubble would occupy the entire length of the tube when the tube was in a horizontal position or when it was slightly inclined to the horizon, thus rendering it impossible to tell when it was in a truly horizontal position. The tube is therefore ground on the inside, so that a longitudinal section is a segment of a circle, as shown in Fig. 28(*b*).

Sensitiveness. A line tangent to the circular arc at its highest point, as indicated by the middle of the bubble, or a line parallel to this tangent will always be horizontal. The line tangent to the center point of the tube, Fig. 28(*b*), is known as *the axis of the bubble tube*. Bubble tubes usually have divisions about ⅛ inch apart and etched in the glass as shown at X and Y, Fig. 28(c). If the tube is moved until the bubble is bisected by Y, a tangent at Y makes an angle α with the axis of the bubble tube. The value of α for a movement of one division is known as the *sensitiveness* of the bubble tube. Ordinary engineer's levels have 20-second bubbles, that is, those requiring a change of angle of 20″ to make a movement of 1 division of the bubble; more accurate levels have 8- or 10-second bubbles; plate bubbles on a transit are 40-second; and the transit long bubble is 25-second.

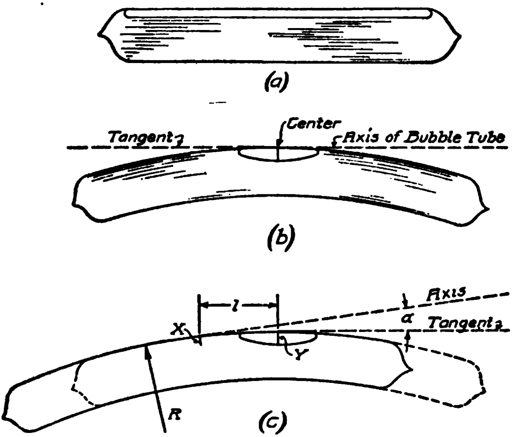


Fig. 28. Diagrammatic Illustration of Level Bubble Tube

Curvature for Bubble Tube. To determine the radius of curvature of the tube, proceed as follows: Let l equal the length and R the radius of curvature of the arc over which the bubble moves for an inclination of l''. Then, from the relation $l : 2\pi R :: l : 360°$, *the radius of curvature* is

$$R = 206{,}265 \times l$$

or the *length of arc* is

$$l = \frac{R}{206{,}265}$$

The radius R may be found by trial. Bring the bubble to the center and sight to a divided rod; raise or lower one end of the level and again sight upon the rod. Call the difference of the readings h, the distance from the bubble to the rod d, and the length through which the bubble moved l. Then, from approximately similar triangles, the *radius of curvature* is

$$R=\frac{dl}{h}$$

Example. In taking measurements with a level bubble as just described, d was 100 feet, h was 0.02 foot, and l was 0.01 foot. What is the radius of the bubble tube? Ans. 50 feet

Adjustment of Bubble Tube. Level bubbles are placed usually in a brass tube, so that only the upper part is exposed, and are firmly cemented in with plaster of Paris. The tube is held to the instrument by screws, as shown in Fig. 29, the end A being fixed and the end B being so arranged that it can be raised or lowered by means of the capstan nuts which are turned by a small steel adjusting pin that fits into the holes in the nut.

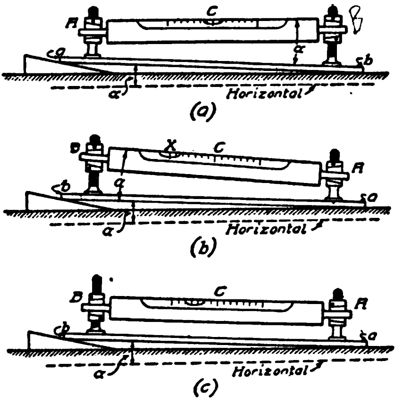


Fig. 29. Leveling with Machinist's Level by Method of Reversion

Reversion Method. The principle of reversion is fundamental in the adjustment of all level tubes. It can be well illustrated in connection with the machinist's level shown in Fig. 29. This level consists of a tube AB attached to a metal base ab by two binding posts as shown. In order to use this level, it is necessary that the axis of the bubble tube and the bottom of the plate ab shall be parallel. If this is true, we know that the axis of the bubble tube will be horizontal when the bubble is in the center, and hence the bottom of ab will be horizontal.

To make the test, set the level on any firm surface and wedge up one end until the bubble is in the center, as shown in Fig. 29(a).

The axis of the bubble tube is horizontal, but suppose *ab* is not parallel to it but makes an angle a with it and is therefore out of the horizontal by an amount a. Reverse the bubble tube, as shown in Fig. 29(*b*), taking care to place it on the same supporting point. The bubble now runs up to X at the end of the tube, the axis of the tube being out of the horizontal by an angle $2a$. The distance XC that the bubble is out from the center of the tube is therefore proportional to $2a$.

If the adjusting screws at B are lowered until the bubble comes halfway back to C, as shown in Fig. 29(*c*), the axis will have been moved through angle a and will then be parallel to *ab*, as required. This adjustment by reversion will be referred to again in discussing the level, transit, etc.

LEVELING PROCESS

Methods of Leveling. Several methods are used for leveling: leveling with the barometer; trigonometric leveling; and leveling with the level bubble. In leveling for foundations, a piece of hose filled with water can be used.

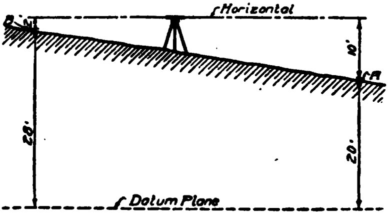


Fig. 30. Diagram Illustrating Leveling Principle

Datum Plane as Basis. The fundamental principle in ordinary leveling is shown in Fig. 30. Suppose that point A is 20 feet above the *datum plane*, which is any convenient level, frequently sea level, but generally arbitrarily selected so that all points in the survey are above it, what is the elevation of point B above the same level? An instrument by means of which a horizontal sight may be determined is placed between the points A and B and readings of 10 feet and 2 feet, respectively, are taken on A and B. It is evident from Fig. 30 that the elevation of B is $20+10-2$, or 28 feet.

In order to do leveling of this kind, a leveling instrument and a rod are needed.

Use of Leveling Rods

Function of Rod. Leveling rods are used in measuring the vertical distance between the horizontal plane through the line of sight and the point upon which the rod is held. These rods are made of hard wood and, when closed, are usually 6½ feet long but slide out to a maximum length of 12 feet. They are provided with target, vernier, and clamps.

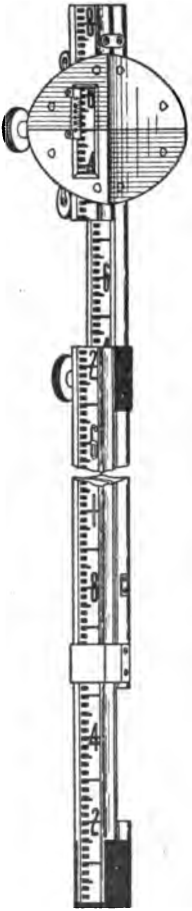


Fig. 31. Philadelphia Self-Reading Rod

Leveling rods are of two kinds, the *self-reading* and the *target* types. Of the self-reading rods, the Philadelphia, and of the target rods, the New York, are in common use. There are, of course, other types, but the two mentioned are the most popular.

Philadelphia Self-Reading Rod. The self-reading rod, Fig. 31, is usually graduated to 100ths of a foot and can be read by the observer directly from the instrument without the aid of the target, as is suggested by the name. With the aid of the target, this rod can be read to 1000ths of a foot. The target is used when greater accuracy is required and when the rod is so far from the instrument that it cannot be read distinctly.

The self-reading rod consists of a graduated scale divided into feet, 10ths of a foot, and 100ths of a foot. Each alternate 100th space is filled in solid with black on a white background, so that the lower side of the first black space is called one hundredth, the upper side of the same space is called two hundredth, the lower side of the third space is called three hundredth, and so on until the tenth hundredth is read.

Reading without Target. When the rod is used as a self-reading rod, that is, without the target, it is extended to its full length—12 feet—and clamped. Readings are then taken on the scale on the front side of the rear half, Fig. 31, which is a continuation of the scale on the front half when the rod is fully extended.

Operation of Rod with Movable Target. The movable target, which is used for close work, has a vernier which reads to 1000ths of a foot and is read from zero to ten. To read this rod, read first the position of the zero of the vernier on the scale, and then add to this the vernier reading found by noting where a line of the vernier coincides with a line of the scale. For example, if the zero of the vernier is just above one foot, four 10ths, and five 100ths, as shown in Fig. 32, and a line of the graduation of the vernier coincides at 7 with the top of a horizontal black line on the rod, the reading will be 1.457 feet. If reading to the nearest 100th, the reading will be 1.46 feet.

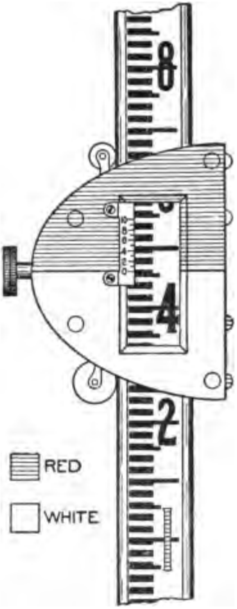


Fig. 32. Reading of Vernier on Philadelphia Rod.

These readings apply only to the face of the rod, or to 6½ feet. When it is necessary to make the rod reading greater than 6½ feet and the target is to be used, the procedure is as follows: The target is clamped on to the rod at the 6½-foot mark. (Attention is called to the fact that the front half of the rod is divided, a short section at the top being fastened permanently to the rear half and carrying the target This is shown clearly in Fig. 31.) The clamp screw holding the two halves of the rod is loosened, allowing the rod to be extended to a maximum of 12 feet. It is extended until the target is bisected by the cross hairs of the instrument, and the height of the target above the bottom of the rod is then read by means of the *a* scale and vernier on the back of the rod. In the rear quartering view, Fig. 33, is shown the front half of the rod *A*, the clamp and vernier *C* attached to *A*, and the extended portion *B*, at the top of which the target was set.

It will be noticed that the back *B* of the rod is graduated from 6.5 at the top down to 12 at the bottom and that the vernier also reads downward. When the rod is contracted to 6½ feet, the zero of the vernier coincides with the 6.5-foot mark on the *B* scale; hence, if the target is set at 6.5 feet on the front scale, the back vernier reads its height when the rod is extended. The rod reading for Fig. 33 is 6.751 feet.

Reading with Target. In taking readings of the rod in leveling, the following procedure should be noted when a target is used: The leveler (at the instrument) should first read the rod through the telescope of the instrument and then notify the rodman at what distance the intersection of the cross hairs in the instrument approximately bisects the rod, such as 3.21, which means three feet, two 10ths, and one 100th. The rodman then sets his rod to read this distance, and another sight is taken, being careful to have the rod plumb. Should the intersection of the target fail to coincide with the cross hairs in the instrument, the leveler then signals the true rod reading—or calls out, if sufficiently near to do so, as up one 10th, down two 100ths, as the case may be—and the target is placed at this distance. The above method is relied upon only for the approximate placing of the target. When precision is required, the method used is to stand behind the rod, holding it between the thumb and fingers of one hand while moving the target with the other hand to the signals of the observer.

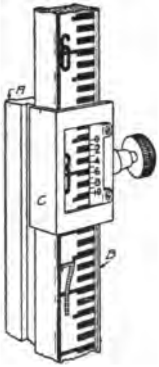


Fig. 33. Back Vernier of Philadelphia Rod

When a slow motion with the hand above the shoulder or below the hip is made by the observer, it means that the rod is to be moved in that direction a fractional part, as one 10th, but when a quick motion is made and the hand drawn back in the same manner, it implies that the target is to be moved just a trifle. In this way and by proper attention to the signals of the observer the rodman can become an efficient and helpful assistant and save much time. When the target is finally set, the rodman reads the rod and calls out the reading to the observer when within reasonable distance, so that the observer may complete his notes. The target rods are read entirely by the rodman and the readings are kept by him in a notebook for that purpose; these notes should be given to the observer at every opportunity and the results should be checked To obtain correct results when leveling, it is absolutely essential that the rod be vertical, and the rodman should remember to hold the rod in that position.

Use of Angle Target. The observer or leveler, by means of the vertical wire in the leveling instrument, can tell whether or not a rod is in a vertical plane through the direction of his sight, but he is not able to detect the error when the top of the rod is inclined toward, or away from, the instrument. The invention of the angle target obviates this trouble, the rod being vertical only when the line of sight coincides with the red and white horizontal boundary lines on both the front and the angle plates, as can readily be seen from Fig. 34. The American target, Fig. 35, fulfills the same requirements by having two discs, one behind the other. The principle of construction of this target is extremely simple and may be best explained in the illustrations shown. Suppose a target of the old kind, which in its front view looks exactly like the front view of the new target, Fig. 35(*A*), to have been cut along the vertical lines *aa* and *bb* and thus divided into three parts, one center piece and two wings. Suppose, furthermore, that the center piece remains in its former place at the front of the rod, while the two wings are removed to the rear of the rod, Fig. 35(*B*). Then the result evidently will be that the horizontal line *cc*—*dd*, Fig. 35 (*A*), will appear to the observer as one unbroken line only when the rod is held perfectly vertical. Any deviation, either toward the instrument or away from it, Fig. 35(*C*), will cause the two parts *cc* and *dd* of the horizontal line situated at the rear of

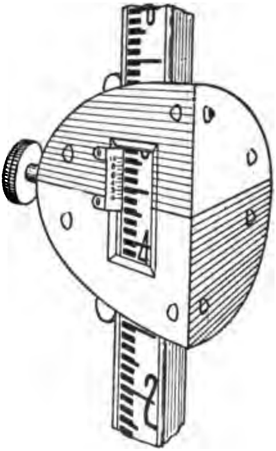


Fig. 34. Angle Target for Setting Rod Vertical

the rod in the wings of the target to show either above or below that part *cd* of the horizontal line which is situated in the front of the rod in the center piece of the target, Fig. 35 (A_1).

Whether using the angle target, the ordinary target, or no target at all, it is apparent that the rodman should hold the rod in a vertical position, known as *plumb*. This can be done by standing directly behind the rod with both feet together or apart, as the rays of the sun-governing shadows may require and, holding the rod between the thumb and finger of one hand, so balancing the rod that it will almost stand in a vertical position by itself. The rodman should never put his hands around the rod.

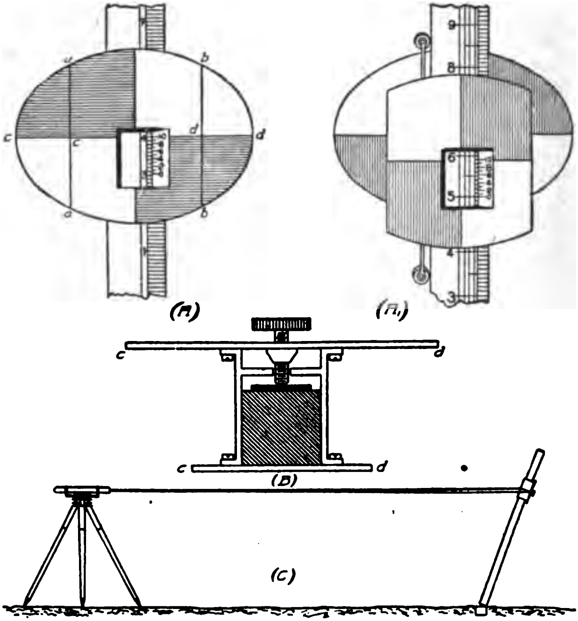


Fig. 35. American Target for Setting Rod Vertical

Rod Signals. When the rod is to be read without the aid of a target or with the ordinary target, it frequently happens that the rod is not vertical, and the signal used for bringing it into its proper vertical plane is the raising of either the right hand or the left hand to a vertical position, which indicates that the rod should be inclined in that direction. In so doing, move the rod slowly until the hand is lowered. After the target has been set in its proper position, clamp it by the screw on its side, then give a check sight.

The observer should use but one hand in signaling the rod. If the target is to be lowered, he should hold his hand below his hip, palm down. To raise the target he should hold his hand above his shoulder, palm up. Any considerable change in the position of the target is denoted by a more or less violent motion of the hand. If a very slight change is desired, the observer should hold his hand in the proper position without moving it up or down. When the proper position of the target has been obtained, the observer indicates the fact by raising both arms above his head and moving them in the arc of a vertical circle to indicate that the rod reading at that particular place is complete.

New York Target Rod. The New York rod, Fig. 36, resembles the Philadelphia rod as to use and dimensions but differs as to scale and vernier reading. The scale is divided into feet, 10ths of a foot, and 100ths of a foot, the same as the Philadelphia rod, except that the graduations of the 100ths, instead of having the sides of one black space each bounding a 0.01-foot distance, as on the Philadelphia rod, are distinct by themselves, and therefore each line between the 10ths represents a 0.01-foot division on the scale. In Fig. 37 is shown a sectional part of the rod with its movable target set at 6½ feet, the black horizontal lines each indicating 0.01 foot, as has been explained. Fig. 38 shows the side, with its graduations as on the face of the rod, set at 6½ feet.

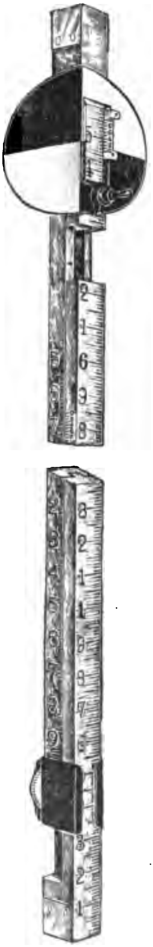


Fig. 36. Keuffel & Esser Type of New York Rod

Use of Target. The New York rod has a movable target which carries a vernier and allows readings to 1000ths. It cannot, however, be read without the aid of the target and is usually made in a cheap form, frequently graduated in feet and inches and having simply an auxiliary scale—which looks like a vernier but should not be confused with it—for closer readings, which is used in construction work. For elevations up to 6½ feet the target is used by sliding it up or down upon the rod in the same manner as with the other type of rod. Above 6½ feet, as with all the rods, the target is clamped at the 6½-foot division. The back of the rod slides upon the front half and, when it is so extended, the vernier is on the side,

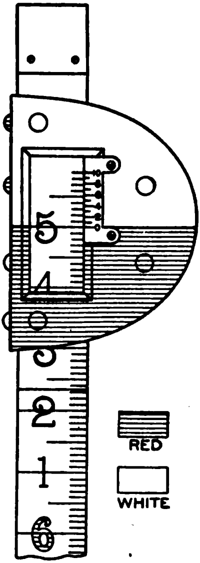


Fig. 37. Vernier Reading for New York Rod

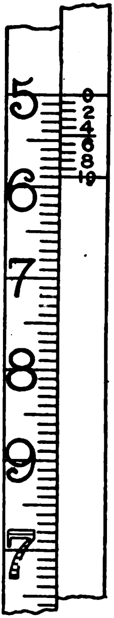


Fig. 38. Vernier on Side of New York Rod

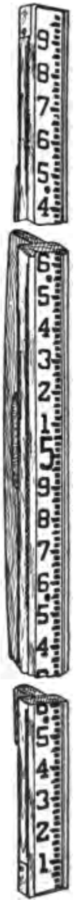


Fig. 39. One-Piece Rod

impressed in the wood, Fig. 38. The method of using the clamps, setting the target, etc., is the same as that for the Philadelphia rod.

One-Piece Rods. In Fig. 39 is shown a rod which is made in lengths of 10 or 12 feet or more with graduations like the Philadelphia rod. The face of the rod is in one piece and is reinforced by a tapered piece of wood fastened to the back which usually has a hole through it for convenience in carrying the rod. The rod has, therefore, a T-shaped cross-section which is very strong. No target is used with this rod as it is a self-reading rod. In order to plumb the rod, a small level bubble of the circular type is fastened to the back of the rod by a hinge so that it may be folded up and will not be liable to break when the rod is not in use.

USE OF LEVELING INSTRUMENTS

Telescopic Sight. Before discussing the leveling instrument, it will be necessary to point out briefly the details and parts of the telescope as used for surveying instruments.

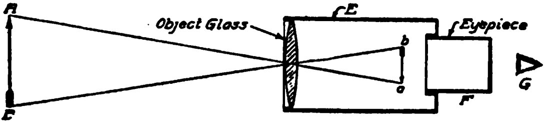


Fig. 40. Diagram Illustrating Principle of Ordinary Field Glass

Principle of Ordinary Field Glass. In Fig. 40 is shown the ordinary field or opera glass. It consists of a large tube E having a lens called the *object glass* in its front end and being fitted at the other end with a smaller tube F that may be moved in and out by a sliding movement. Suppose we look at any object AB with this telescope. A real image of the object is formed by the objective, or object glass, inside the tube E but inverted as shown at ba. This image is smaller than the object, and in order to magnify it we use the tube F which holds one or more lenses and is called the *eyepiece.* If only one lens is used in F, the image ab is magnified but is seen by the eye at G still in the inverted position; this is an *inverting eyepiece.* If the proper combination of lenses is used in the eyepiece, the image ab is not only magnified but is brought back to its proper position, and the eyepiece is called an *erecting eyepiece.*

Construction of Telescopic Sight. A study of Fig. 41, together with Fig. 40, will make clear to the student the principles involved in the telescope on a surveying instrument. It is really more than a telescope; it is a telescopic sight. From Fig. 41 it will be seen that a telescopic sight differs from a simple telescope, Fig. 40, in that the tube F is made in two parts which slide in and out and in having, in addition, cross hairs placed where the image ab, Fig. 40, is shown. The construction of the cross-hair ring is shown in Fig. 41(a). Four oval holes are cut through the telescope tube and a metal ring, quite a little smaller than the inner diameter of the tube, is held in position by the tension of four capstan-headed screws that pass freely through the slotted or oval holes in the tube and fit in threads cut in the ring. These screws have washers under their heads which cover the slotted holes and enable the screws to be brought to a bearing against the tube. Fastened to the ring, as shown, are the cross hairs, there being one vertical cross hair and one or more horizontal ones. These hairs are usually spider webs—fine platinum wire is also used—and are held in place by a little shellac. It is evident that by loosening all the four screws, the cross-hair ring becomes loose in the tube and may be rotated slightly within the limits of the oval holes through the tube. Also, by loosening the upper screw and tightening the lower, the ring may be moved toward the bottom of the tube and in a similar manner upward or to the left or the right. This should be clearly understood, as it is important in the adjustment of instruments.

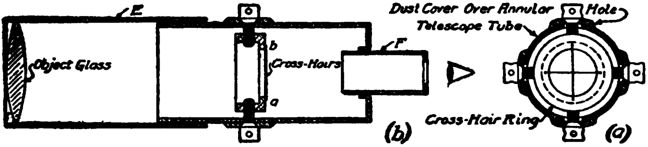


Fig. 41. Section of Telescope Showing Object Glass, Eyepiece, and Cross-Hair Mountings

Operation of Telescopic Sight. Returning now to the operation of the telescopic line of sight, it is evident that the cross hairs and the image formed by the object glass will not both be in focus when viewed through the eyepiece unless they are at exactly the same point. As the position of the cross hairs is fixed, the front portion of the tube E, and hence the object glass, Fig. 41 (b), is made to move so that the image is formed in the plane *ab* of the cross hairs. This adjustment cannot be made independently, however, but must be accomplished through the eyepiece in two steps, as follows:

(1) The eyepiece must be focused on the cross hairs. This can best be done by turning the telescope toward the sky and moving the eyepiece in or out until the cross hairs appear sharp and clear. This adjustment ordinarily need be made but once by an instrument man, but another observer would probably have to change it.

(2) We are now in a position to move the objective slide in or out, that is, to focus it, so as to make the image clear and distinct; when this is done, the image must be in the plane of the cross hairs. As a test of this, see whether the cross hairs change their position on the object sighted at when the eye is moved to different points of the opening at the end of the eyepiece. Such motion of the cross hairs, known as *parallax*, proves that the cross hairs and the image do not fall in the same plane, owing to poor adjustment of the eyepiece or to poor focusing of the objective. Readjusting for both (1) and (2) will bring a perfect agreement. The eyepiece now magnifies not only the image of the object but also the cross hairs, and the cross hairs appear as black lines projected on the object. The line from the instrument to the place on the object where the intersection of the cross hairs appears to be is the line of sight and is the prolongation of the line through the center of the cross hairs and the center of the objective.

Principle of Instrument Leveling. The leveling instrument consists of a telescopic sight combined with a delicate level bubble. It will be evident that the axis of the bubble tube is horizontal when the bubble is in the center of the tube, and if the line of sight in the telescope is made parallel to this axis, the line of sight will be horizontal, and we shall have a very accurate means of determining the difference in elevation of two or more points. Levels are of three kinds: the wye, the dumpy, and the precise.

Mechanism of Wye Level. *Sight and Bubble.* The wye level is shown in Fig. 42. It consists of two parts held together by clamps. The upper part, shown in cross-section, Fig. 43, is simply a telescopic sight with a level bubble fastened to the lower side of the tube by adjustable screws in exactly the same manner as the machinist's level, Fig. 29, is attached to its base. The objective is focused by turning the ratchet screw *C*, Fig. 42; the capstan screws shown at *D* control the cross-hair ring, etc.; and the eyepiece is focused by the screw *E* or by simply turning the eyepiece end *F* when there is no such screw

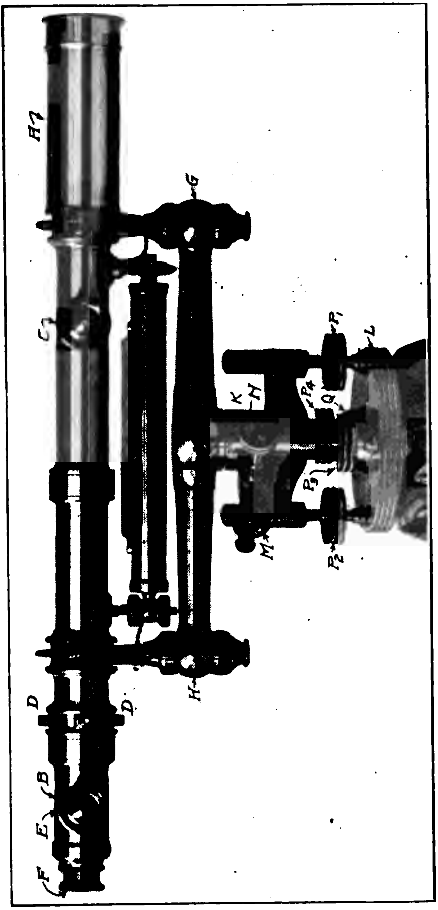


Fig. 42. Typical Wye Level

Courtesy of W & L. E. Gurley Company, Troy, New York

on the instrument. As previously stated, the eyepiece must be properly focused or the cross hairs may not be visible at all.

Wyes and Leveling Head. The lower part of the instrument is a support for the telescope and an arrangement for leveling the instrument. This consists of a horizontal bar GH having at each end a Y-shaped support from which the instrument gets its name. The telescope is held in these supports by two clamps, as shown. Fastened to the center of the horizontal bar is a vertical axis K held by the leveling head L which screws on to the top of the tripod, Fig. 49. When free, the entire instrument may be turned about the vertical axis K, or it may be clamped in any position by the clamp screw M and then turned slowly by the slow-motion screw N. These latter screws are seldom used and are not really needed on the instrument. The leveling head consists of a 4-arm projection O which is fastened to the socket in which the vertical axis fits and is held away from the plate L by four leveling screws P_1 and P_2, etc. The lower end of the socket has a circular bearing Q holding it to the plate. It will be evident from this description that the entire instrument is leveled up by moving the leveling screws. Thus the telescope is turned until it is over the two diagonally opposite screws P_1 and P_2, and the bubble is brought to the center by turning these screws; the telescope is then turned 90 degrees so as to come over the two other screws and is again leveled. In operating these diagonally opposite screws, it should be noted that one must be shortened, that is, screwed upward into the part O, in order that the other may be lengthened, thus tilting the instrument. Both these things are done at the same time, one hand being used on each screw. Thus, to raise the end B the instrument, P_1 is lengthened and P_2 is shortened by turning the screws "thumbs in", and *vice versa.*

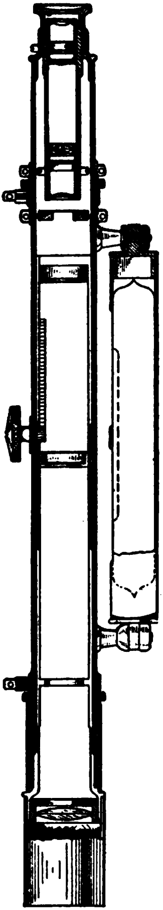


Fig. 43. Cross-Section of Typical Telescope on Surveying Instrument

Reversion Method of Adjusting Wye Level. The wye level may be adjusted by the method of reversion and by the peg method. The peg method will be described later as it is the only method of adjusting the dumpy level; it should be used in adjusting an old wye level in which the wyes are worn or as a check on the reversion method. In the reversion method there are two adjustments to a level: making the axis of the bubble tube and the line of sight parallel, so that the line of sight will be horizontal when the bubble is in the center of the tube; and adjusting the vertical axis of the instrument. These adjustments cannot be made permanently by the maker as they are too delicate to remain long in adjustment and are liable to be disturbed by a knock or a jar.

Adjusting Parallelism of Axis of Telescope and Level Bubble. This first adjustment is essential to a proper use of the instrument and is made in two steps: (1) adjusting the line of sight parallel to the bottom of the collars and hence parallel to the wye supports; (2) adjusting the bubble tube by making the axis of the bubble tube parallel to the wye supports and hence parallel to the line of sight.

(1) Adjustment of Line of Sight. To make this adjustment, set up the instrument firmly upon solid ground, shaded from sun and wind. Remove the pins holding the telescope clips and raise the clips. Direct the telescope toward the side of a building, a fence, or other convenient object and carefully move the instrument until the intersection of the cross hairs is upon a well-defined point, such as the head of a tack. Clamp the vertical axis by means of the screw *M*, Fig. 42, and slowly revolve the telescope in the wyes, noting if the intersection of the cross hairs continues to cover the point. If so, the line of sight is in adjustment, that is, it coincides with the axis about which the telescope is rotated, which is an imaginary axis through the center of the collars, or rings, which bear on the wyes. These collars being of exactly the same diameter, the line of sight must be parallel to their bottoms and hence parallel to the wye supports.

If the intersection of the cross hairs moves off the point, revolve the telescope in the wyes as nearly as possible through 180 degrees and carefully mark a point at the intersection of the cross hairs in this last position. Bisect the distance between the two points, establishing a center, or third point, which is the point where the axis of rotation of the telescope prolonged would come. By means of the capstan screws D, Fig. 42, attached to the cross-hair ring, move the ring so that the intersection of the cross hairs covers this point. Now repeat the test and correct the position of the cross-hair ring until the intersection of the cross hairs stays on one point as the telescope is revolved in the wyes.

When making this adjustment, an experienced instrument man will also test and adjust the vertical wire. The object is to make the vertical wire truly vertical. It is accomplished by making the horizontal wire, which is placed on the ring by the maker at 90 degrees to the vertical, perpendicular to the vertical axis. When the vertical axis is vertical, then the horizontal wire will be truly horizontal and the vertical wire truly vertical. In making the above adjustment, start by setting the line of sight on the first point with the telescope turned 180 degrees from its normal position, that is, with the bubble on top. Rotate the telescope 180 degrees to its normal position and test this position by noting whether the wye clamps will fit properly in position. Mark the point where the intersection of the cross hairs now comes and find the center as before. Now adjust the intersection of the cross hairs so it bisects this point and, if necessary, twist the cross-hair ring slightly, loosening all screws, *so the horizontal wire will also stay on this point* when the instrument is turned slightly about its vertical axis. The two adjustments are thus made at one time. It will be noticed that to make this adjustment, it is not necessary to level the instrument.

(2) Adjustment of Axis of Bubble Tube. To adjust the level bubble, first loosen the clips holding the telescope in the wyes; then revolve the telescope upon the vertical axis to bring it directly over a pair of leveling screws and clamp it by means of the screw M, Fig. 42. By means of the leveling screws P_1 and P_2, Fig. 42, bring the bubble to the center of the tube as accurately as possible. Now, without disturbing the instrument, carefully lift the telescope out of the wyes and turn it end for end, being careful when it is replaced in the wyes that the telescope comes to its proper seat at each end. If the bubble in this new position of the telescope comes to rest at the center of the tube, the axis of the tube is in adjustment.

If the bubble does not return to the center, bring it halfway back to the center by means of the adjusting screws at one end of the bubble tube and the remainder of the way by the two leveling screws. Now repeat the test and correction as often as is necessary until the bubble remains in the center of the tube.

The theory of this adjustment, Fig. 44, is simply the application of the principle of reversion, Fig. 29. Fig. 44(a) shows the telescope and level bubble resting in the wye supports (shown by arrows) leveled so the bubble is in the center of the tube. The first adjustment made the line of sight coincide with the axis of the collars and hence parallel to the line AB through the bottom of the collars. It now remains to make the axis of the bubble tube ab parallel to this same line AB. Suppose the angle of error is a, as shown. When the telescope is removed from the support and turned end for end, as shown in Fig. 44(b), the line AB does not change but is still out by angle a from the horizontal, as it was in Fig. 44(a). The axis of the bubble tube will hence be $2a$ from the horizontal, or the distance xc represents $2a$. Hence, if we adjust the screws at the end b so as to lengthen this support and bring the bubble back halfway, we shall have moved the axis of the tube through the angle a, and its axis ab will be parallel to AB and hence parallel to the line of sight, as required.

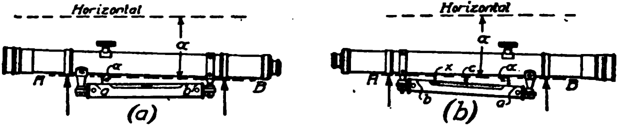


Fig. 44. Diagram Showing Theory of Adjustment of Bubble Tube Axis

Most instruments have what are called *lateral adjusting screws* attached to one end of the bubble tube. The adjustment we have just made results in bringing the axis of the bubble tube and line of sight in parallel planes. These lateral adjusting screws are used to shift one end of the tube in or out so that its axis will be in the same *vertical plane* as the line of sight. Such an adjustment need seldom be made and is omitted on most instruments but, when necessary, it is made by trial as follows: Loosen the clamps on the wyes and rotate the telescope slightly from the vertical position. If the bubble stays at one point in the tube, it is all right. If it moves, it will move to the high end of the tube, which must be adjusted accordingly by loosening the lock nut on the proper end of the tube and then using the lateral screws. This should be tested and adjusted before making adjustment (2) as it would derange that adjustment.

It must be borne in mind that the adjustments (1) and (2) must be carefully made and that they are absolutely essential if satisfactory results are to be attained with the instrument.

Adjustment of Vertical Axis. This adjustment, sometimes called the *wye adjustment*, is not absolutely essential provided that every time a reading is taken the bubble is brought to the center of the tube by means of the parallel plate screws. However, the adjustment will expedite field work and should always be made.

To make the test, level the instrument carefully over both pairs of leveling screws; if the bubble remains in the center of the tube as the telescope is revolved on the vertical axis all the way round, the adjustment is complete. If the bubble runs to one end as the telescope is thus revolved, the vertical axis is out of adjustment and may be corrected as follows: Bring the telescope directly over a pair of opposite plate screws and by means of these screws bring the bubble accurately to the center of the tube. Now revolve the telescope on the vertical axis as nearly as possible through 180 degrees and note the displacement of the bubble; bring the bubble halfway back to the center by adjusting the large capstan screws which fasten one of the wyes *H*, Fig. 42, and the remainder of the distance by means of the leveling screws P_1 and P_2, Fig. 42. Repeat the test and adjustment until the bubble remains in the center of the tube in all positions of the telescope.

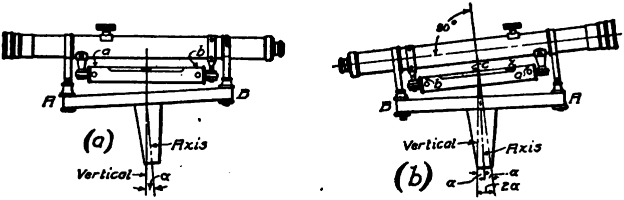


Fig. 45. Diagram Showing Adjustment of Vertical Axis of Instrument

This adjustment is also based on the principle of reversion and is illustrated in Fig. 45. In Fig. 45(*a*) is shown the first position with the instrument leveled and the bubble in the center of the tube, supposing the vertical axis to be out from the perpendicular to the axis of the bubble tube by the amount of the angle a. When the instrument is revolved through 180 degrees about its vertical axis, the conditions are as shown in Fig. 45(*b*). The vertical axis stays the same, but a perpendicular to the bubble-tube axis ab makes an angle a to the right or a total angle $2a$ with the vertical. The bubble runs up to the end a of the tube and the distance cx represents the angle by which the bubble-tube axis is out from the horizontal, or $2a$. If the wye at A is shortened by means of the two large capstan screws until the bubble comes halfway back to c, the axis of the tube will have been moved through the angle a, and a perpendicular to it will then coincide with the vertical axis, as required.

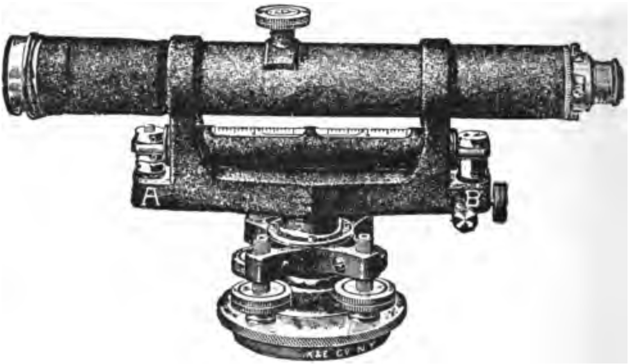


Fig. 46. Typical Dumpy Level
Courtesy of Keuffel & Esser Company, New York City

Replacement of Cross Hairs. *Material.* The cross hairs in leveling instruments may be either spider webs or platinum wire; spider webs are better. Probably the web of the little black spider is the most satisfactory, but the web of the common spider will give good results, especially if freshly spun.

Method. To replace the cross hairs, carefully remove the cross-hair ring, Fig. 41, from the telescope tube and lay it upon a white surface with the cross-hair side upward. Carefully clean off the material used to fasten the hairs to the ring and it will be noticed that there are incisions upon the face of the ring intended to indicate the position of the cross hairs. A portion of the web is then taken between the points of a pair of dividers and, after being slightly stretched, is placed in position over the marks and the ends of the spider web are fastened to the ring with a touch of shellac, varnish, or wax.

Characteristics of Dumpy Level. The dumpy level, Fig. 46, differs from the wye level in the following points: There are no wyes, the telescope being firmly attached to the top of the uprights which, in turn, are attached firmly to the bar AB. The level tube also is attached by the usual screws, not to the telescope but to the bar AB. The dumpy level is not so convenient in its adjustments as the wye level; nevertheless, when properly adjusted, it will give equally good results and, on account of its simplicity and compactness, it is not so liable to have its adjustments disturbed.

Adjustment of Dumpy Level. The adjustments of the dumpy level are similar to those of the wye level but are made in the reverse order. They are two in number: adjusting the axis of the bubble tube perpendicular to the vertical axis of the instrument; and adjusting the line of sight parallel to the axis of the bubble tube.

Perpendicular Adjustment of Bubble Tube. The theory of the first adjustment is identical with that of the adjustment of the vertical axis of the wye level. The adjustment of the dumpy is made, however, by means of the adjusting screws at the end of the bubble tube, not by the wye screws. Thus, to make the first adjustment, set up the instrument firmly in a position shaded from sun and wind. With the telescope over a pair of opposite leveling screws, bring the bubble accurately to the center of the bubble tube by means of these screws. Repeat the operation over the other pair of leveling screws.

Now turn the telescope accurately through 180 degrees to the position directly over the first pair of leveling screws. If the bubble remains in the center of the tube, the adjustment is complete. If the bubble does not remain in the center of the tube, bring it halfway back to the center by means of the vertical capstan screws at one end of the bubble tube and the remainder of the distance by the leveling screws. Now repeat the test and the adjustment until the bubble remains in the center of the tube through all positions of the telescope.

Adjustment of Line of Sight by Peg Method. The second adjustment of the dumpy level must be made by the peg method, as the telescope cannot be removed from the wyes. This method consists essentially in determining a horizontal line and in moving the cross hairs, if necessary, so that the line of sight is on this line when the axis of the bubble tube is horizontal; that is, when the bubble is in the center of the tube.

Select a piece of ground as nearly level as possible and lay out a straight line upon it, from 300 to 400 feet in length, driving a stake at each end and at the center. Set up the level over the center stake, Fig. 47(*a*), and, after leveling, carefully direct the telescope to the rod held upon the stake *N* and take the reading by the target. Now

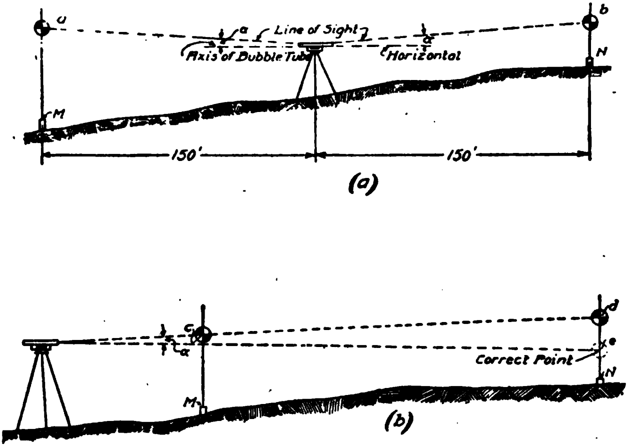


Fig. 47. Diagram Illustrating Adjustment of Line of Sight of Dumpy Level by Peg Method

direct the telescope to the rod held on the stake *M*, see that the bubble is in the center of the tube, and take the reading. The difference of the two rod readings at *N* and *M* will be the true difference in elevation, no matter how much the line of sight is out of adjustment. This is shown in the figure, where it is seen that the line of sight, if not parallel to the axis of the bubble tube, makes an angle *a* with it, and that *a* will be the same for both pointings if the bubble is in the center of the tube. Furthermore, as the instrument is *halfway* between *M* and *N*, both rod readings will be affected the same amount.

Now remove the instrument to some point about 15 feet back of either *M* or *N*, as shown in Fig. 47(*b*). Level up, take the rod readings *c* and *d* at *M* and at *N*, respectively, and find the difference. If this difference agrees with that first found, the instrument is in adjustment. If it does not, the correct reading *e* of a rod upon *N* must be computed. In the case shown in Fig. 47 the correct reading *eN* will be almost equal to *cM* minus the true difference in elevation. The target must now be set at this point *e* and, by moving the capstan screws holding the cross-hair ring, the line of sight must be adjusted until it bisects the target, care being taken to keep the bubble in the center of the tube. If the instrument is placed back of *N*, the difference in elevation must be added. Note that this is only approximately the proper reading. It is in error a slight amount owing to the fact that *c* is slightly above the point where a horizontal line from the instrument cuts the rod on *M*, caused by the instrument being out of adjustment. After the first trial adjustment of the instrument, take another reading at *M*, compute the new reading *eM* as before, and test and make perfect the adjustment.

It has been noted that the second adjustment of the wye level may be made by the peg method, but the reversion method given is the more convenient and is used, unless the collars are worn.

Precise Level. One form of the precise level is shown in Fig. 48. This instrument is the same in principle as the wye or the dumpy level, but it has two additional attachments which add greatly to its convenience and accuracy. It is leveled up in the same manner as the wye or the dumpy levels, except that a leveling head with only three screws is used. When the sight is taken, however, the observer may glance upward to the mirror *M* which reflects the position of the bubble in the tube *T*—in the type shown this is placed over the telescope—and, by means of the micrometer screw *S*, which raises or lowers one end of the horizontal bar *AB*, bring the bubble exactly to the center of the tube.

Comparative Usage of Types of Levels. *Precise Instrument.* At the present time the precise level is seldom used in general practice, but although more expensive, it will undoubtedly be more generally used in the future. This is particularly true of the type illustrated, which is really a semi-precise level, with which both greater speed and greater accuracy may be obtained than with the ordinary type.

Wye and Dumpy Instruments. In choosing between a wye and a dumpy level, it is thought that the dumpy is rapidly growing in favor owing to its compactness and rigidity. In foreign countries the inverting eyepiece is used on both transits and levels but in the United States it is generally used only on levels. It offers an advantage in clearness, as it has fewer lenses than the erecting form, and each lens takes up a small portion of the light passing through the

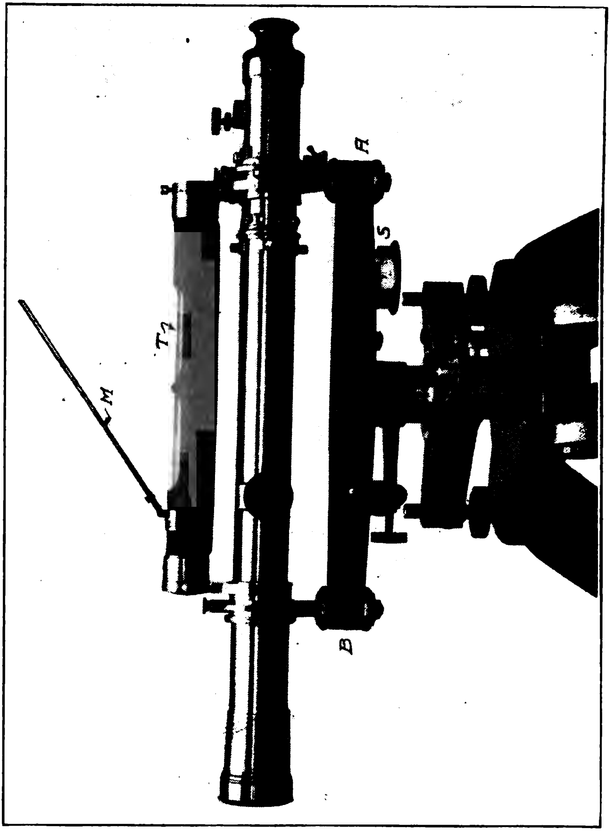


Fig. 48. Typical Precise Level
Courtesy of Keuffel & Esser Company, New York City

telescope. The real disadvantage of the inverting eyepiece is that everything appears upside down; but an instrument man soon becomes accustomed to this.

Method of Setting Up Instruments. The term *setting up the level* means to place it in position to secure horizontal sights. To do this, plant the legs firmly in the ground at approximately equal distances apart so as to make the leveling head about horizontal. Then the instrument is leveled up by turning the telescope so that it comes directly over two diagonally opposite leveling screws P_1 and P_2, Fig. 42, and by bringing the bubble to the center by turning these screws "thumbs in" or "thumbs out", as already described. It is then turned through 90 degrees to a position over the other pair, is leveled in this direction, and as this is liable to derange the first leveling, it is brought back 90 degrees and re-leveled, etc., until it stays within a division of the center for all positions of the telescope. The bubble is in the center when its length on each side of the center, read on the graduations on the tube, is the same. With few exceptions, the level is never placed in line except when being adjusted under the peg method. It is usually placed in some convenient spot where the greatest number of sights can be secured and, in accurate differential leveling, it is placed so as to make the distance between the instrument and the rod on each sight equal. As already stated, the tripod legs

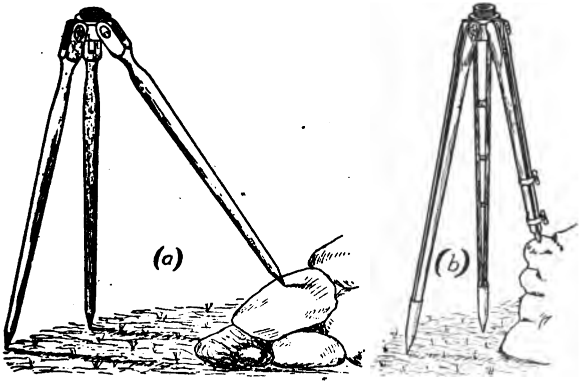


Fig. 49. Method of Setting Tripod on Uneven Ground With or Without Adjustable Leg

must be so placed as to make the leveling head about horizontal. This will save time in bringing the bubble into its proper position. Should it be required to set up the instrument on the side of a hill, place one leg at an altitude and the other two in apparent line with each other, Fig. 49, but where the tripod is adjustable the proper method is apparent. Practice in the field has shown that adjustable legs are not desirable owing to the difficulty in keeping them from slipping.

After the instrument is set up and leveled, focus the eyepiece upon the wires and then focus the object glass on the rod by means of the ratchet screw *C*, Fig. 42. Care should be taken not to take a reading until the bubble has been carefully observed and brought into the exact center of the bubble tube. The care with which this need be done depends upon the accuracy required. *The movement through one division of a 20-second bubble corresponds to about 0.03 foot on a rod 300 feet away.* The readings on the rod are being taken to only 10ths of a foot, and there is no need for leveling up closer than this. On the other hand, for very accurate work, this should be done with great care. When this is completed, sight through the telescope and note the rod reading or set the target rod; again look at the bubble and see if it has moved away from its former position; if not, again sight on the rod and see if the first observation was correct. Should the intersection of the cross hairs fail to coincide with the horizontal and vertical lines of the target or the center of the rod, the rodman is to incline the rod by the signals of the observer until it is vertical.

Care of Instruments. The care of instruments properly belongs to the instrument man or leveler. While in the field, the instrument remains on the tripod and is carried from place to place as the work requires, but when taken any distance, such as on railway trains, street cars, etc., it should be carefully placed in the box and carried by one who is capable of giving it proper care and attention. The instrument man, being responsible for the instrument, it is natural and perhaps best that he should always carry the instrument. In fact, the greatest amount of precaution should be exercised in the care of the instrument, both in the field and in conveying it from place to place.

Instruments in general, and the level in particular, should never be unduly exposed to the rays of the sun, as this will have a tendency to throw the various sensitive parts out of adjustment; therefore, whenever possible and whether or not it is on the tripod, place the instrument in the shade.

The leveler should always exercise great care not to disturb the instrument after it is set up and should avoid, as much as possible, walking around it unnecessarily, especially if the ground is soft or the position of the instrument is not very firm. It frequently is necessary to set up the instrument among loose timber, rocks, etc., where the importance of this care is apparent. If the instrument is disturbed to any great extent, it will be necessary to relevel it, and if the position of the legs of the tripod is disturbed, the entire work must be done over, because the height of the instrument will not be the same as in its former position. Should it be disturbed after a turning point has been established and its elevation ascertained, it will be necessary only to take a reading on the last turning point to determine the new height of the instrument. After leveling, the instrument man should keep his hands off the instrument except for the purpose of leveling and adjusting the telescope. He should not make a practice of leaning his weight on the tripod.

It is often necessary to send instruments great distances, and in no case should they be sent by express or freight without first being properly packed and secured against breakage. The student should appreciate the fact that the care of the instrument is just as important to good work as the original accuracy of manufacture.

"DON'TS" FOR LEVELING INSTRUMENTS

The following "Don'ts" will be of value if observed in field work:

Do not pick up the instrument by the telescope. Grasp it by the wyes.

Do not carry the level, unless unclamped, either on the tripod or on the arm.

Do not stand any flag or rod where it may fall upon the level.

Do not kick the legs of the tripod when moving about the level.

Screw the head firmly to the tripod.

Screw the legs until they will just slip at the head joint; this will eliminate lost motion.

Do not carry plumb bobs loose in the level box.

Do not use the level box as a seat.

Do not use the level box as a storage case.

Do not leave the level box cover unfastened.

Do not knock the box over or place it where it will be subject to shocks.

When carrying the level off the tripod, rest the bar on the arm with the level screw plate in the angle of the elbow.

When carrying the level on the tripod, be careful not to hit the level against any obstruction.

Always remember that the level is a costly sensitive piece of mechanism.

Differential Leveling. The method used to determine the difference in elevation between two points, both of which are visible from a single position of the instrument, has already been described in connection with Fig. 30. The difference between the rod readings gives the difference in elevation of the two points.

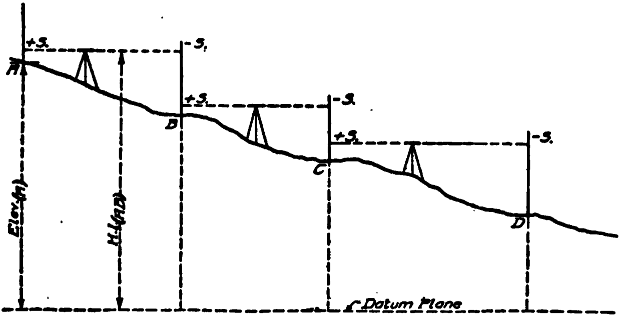


Fig. 50. Diagram Showing Method of Differential Leveling

If the points are too far apart, or if the difference in elevation is too great to be determined from one setting of the instrument, intermediate points must be taken. For instance, suppose it is desired to find the difference in elevation of *A* and *C*, Fig. 50, *C* being too far below *A* to permit readings upon both points from a single position of the instrument. Set up the instrument, not necessarily on the line from *A* to *C*, but in such a position that the line of sight will strike the rod as near its foot as it is possible to take a reading; send the rod to some point (say, *B*) where the line of sight will strike near the top of the rod when it is extended. The difference between these rod readings will give the difference in level of *A* and *B*. Now carry the instrument to such a point that rod readings can be taken upon *B* and *C*. The difference between the rod readings upon *B* and *C* added to the difference between the rod readings upon *A* and *B* will give the difference in level of *A* and *C*, proper attention being given to signs.

Use of Datum Plane. If the line of levels is very extended, the above method of computation is awkward, as some of the differences will be positive and some negative. Therefore, choose such a plane that all points in the line of levels will lie above it; this plane is called a *datum plane*, Fig. 50. Then, beginning at the point *A*, assume the elevation of the point above the datum plane. Read the rod held upon *A*, and the reading added to the assumed elevation will give the height of the line of sight of the instrument above the datum plane, which is called the *height of instrument* (H.I.). Turn the instrument upon the point *B* and read the rod, and it is evident that this last rod reading subtracted from the height of instrument will give the elevation of *B* above the datum plane. Move the instrument beyond *B*, or at least where it can command a view of *B* and *C*, and again sight to the rod held upon *B*. This last rod reading added to the elevation of *B* will give the new height of the instrument, and if the rod reading at *C* is subtracted from this, the elevation of *C* above the datum plane will be given.

Basis of Readings. Referring now to Fig. 50, the first rod reading taken upon the point *A* is ordinarily called a *back sight* and the first reading taken upon *B* is called a *fore sight*. There seems to be no good reason for adhering to this method of distinguishing between the rod readings and it is illogical and misleading. A back sight is not necessarily taken behind the instrument, that is, in a direction contrary to the progress of the survey, neither is a fore sight necessarily taken in front of the instrument. It is more logical and less misleading to designate these rod readings by the terms plus sight and minus sight.

A *plus sight* (+S), therefore, is a rod reading taken upon a point of known or assumed elevation. To determine the height of the instrument, add this rod reading to the elevation at the point where the reading was taken.

A *minus sight* (−S) is a rod reading taken upon a point of unknown elevation. This, when subtracted from the height of the instrument, will give the required elevation.

A *bench mark* (B.M.) is some object of a permanent character, the elevation of which, together with its location, is accurately determined for future reference and for checking the levels.

A *peg*, a *hub*, or a *turning point* (T.P.) is a point used for the purpose of changing the position of the instrument. As a turning point, a firm stone or the top of a spike or stake driven into the ground may be used. It is really a temporary B.M.

Record of Field Notes. If a self-reading rod is used, the instrument man will carry the notebook and record the rod readings as they are observed. The leveler should cultivate the practice of calculating the elevations of his stations as the work progresses, thereby enabling him to discern errors when they occur. If a target rod is used upon the work, the rodman also should carry a notebook in which he should at least enter all readings upon turning points and bench marks, checking up with the instrument man at every opportunity. Under the circumstances, the instrument man is more or less dependent upon his rodman for the correct reading of the rod, and when an inexperienced rodman must be employed, the self-reading rod will give the better results. The limit of range of an ordinary leveling instrument is about 400 feet and sights should not be taken at a greater distance.

The method of keeping the field notes for differential leveling, as outlined in connection with Fig. 50, is given below. A level notebook especially adapted to the purpose should be procured, the notes being entered on the left-hand pages, and the right-hand pages being reserved for remarks, sketches, etc.

Station	+S.	H.I.	−S.	Elevation	Remarks
A	0.650	1000.650	.....	1000.000	*N. E. corner
B	1.250	993.140	8.760	991.890	of abutment
C	2.380	987.670	7.850	985.290	Main Street
D	......	....	9.570	978.100	bridge

These quantities should be checked as often as possible to discover errors in addition or subtraction, making use of the fact that the algebraic sum of all the plus and minus readings equals the difference in elevation of the first and the last stations, that is, from the above figures

$$\begin{aligned}\Sigma(+S)&=+\ 4.280\\ \Sigma(-S)&=-\underline{26.180}\\ \text{algebraic sum}&=-21.90\end{aligned}$$

Adding this to the elevation of station A, we have

$$1000-21.90=978.10$$

This checks with the field notes.

Profile Leveling. The method of profile leveling is the same in principle as above outlined, but the details of field work are a little different.

In this sort of work it is intended to determine a vertical section of the ground above a datum plane. To this end, rod readings are taken sufficiently close together so that when the elevations are plotted and the points connected, the resulting irregular line will closely approximate the actual line of the surface.

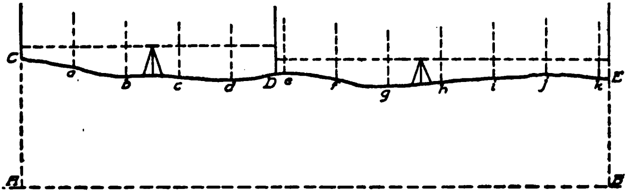


Fig. 51. Diagram Showing Method of Profile Leveling

Profile levels are usually run in connection with a transit or chain survey of the line, the positions of the points being first established. These points are usually 100 feet apart, unless the ground is very irregular, and then they may be 50 or 25 feet apart or even less, the points being indicated by stakes. The readings should be taken with the rod held upon the ground.

In Fig. 51 is shown the difference between profile leveling and the system outlined first and sometimes called differential leveling, or peg leveling. AB is the datum plane, and the full lines at C, D, and E represent positions of the rod for turning points. Assuming the elevation of the point C, the rod is held upon it, and the reading is added to the elevation to determine the height of the instrument. The rod is then carried successively to the points a, b, c, and d, and each reading is, in turn, subtracted from the height of the instrument at *C* to get the elevations of these points.

The rod is then held upon the point *D*, the instrument is moved, and the plus sight upon *D* is added to its elevation for the new height of the instrument. The rod readings upon *e*, *f*, *g*, *h*, *i*, etc., are then each subtracted from this new height of instrument for their elevations.

The improper use of the terms back sight and fore sight is illustrated in Fig. 51; the rod readings at *C*, *a*, and *b* are taken behind the instrument, but the rod reading at *C* is the only plus sight.

The method of keeping the field notes in connection with profile leveling is illustrated below. It will be noticed that the minus sights are placed in two columns. This is done in order that the sights on the turning points may be kept separate from the side shots (S.S.), as only the former are used in making the arithmetical check, the difference in elevation equaling the algebraic sum of the plus and the minus sights.

Station	+S.	H.I.	−S.		Elevation
			T.P.	S.S.	
0	3.25	585.70	....	.....	582.45
+50	.....	......	....	3.78	581.92
1	.....	......	....	4.18	581.52
+50	.....	......	....	5.06	580.64
T.P. +62	2.64	581.49	6.85	.....	578.85
2	.....	......	....	3.10	578.39
+50	.....	......	....	3.18	578.31
3	.....	......	....	3.90	577.59
+50	.....	......	....	4.60	576.89
T.P. 4	.....	......	5.25	.....	576.24

Errors and Accuracy in Leveling. Errors in leveling may be divided into three classes according to their sources: instrumental; natural; and personal. The errors arising from these sources must be studied so that they may be eliminated or guarded against. Instrumental errors, due to poor adjustment, etc., can be eliminated by making the summation of the distances of the plus sights approximately equal to the summation of the distance of the minus sights. This simple precaution also eliminates error from natural sources, such as refraction, curvature of the earth, etc., and it should be followed in accurate work. On hot days it will be noticed that sights close to the ground are rendered difficult by the air boiling or quivering; accurate work in such weather is very difficult, as the sun also causes trouble with the instrument unless it is shaded. Windy weather also renders work difficult.

One of the first steps in leveling is to establish bench marks, and differential levels usually are run in both directions in order to check the elevations. Such work should check back to the starting elevation within an amount equaling a constant times the square root of the distance leveled in miles. This constant for very accurate work (precise leveling) usually is about 0.02 foot to 0.05 foot.

EXAMPLES FOR PRACTICE

1. Given the following notes; complete the computations and check them, using the proper form of notes for profile leveling.

Station	+S.	−S.	Elevation
B.M. 1	10.21	.....	1063.28
0	......	6.20	....
1	.....	7.38	....
2	.....	5.64	....
T.P.	8.97	3.28	....
3	.....	7.52	....
3+50	.....	8.91	....
4	.....	7.11	....
5	.....	6.37	....
B.M. 2	.....	4.39	....

2. A line of levels is run from B.M. 1, the elevation of which is 1092.37 feet, to B.M. 2, a distance of 2 miles, and a check line is run back to B.M. 1 again. Upon reaching B.M. 1, the levels give its elevation as 1092.87, and hence the accumulated error in running 4 miles is +0.50 foot. Does this represent accurate work? Ans. No; it represents an accuracy of $0.25\sqrt{L}$, in which L is the distance in miles. This might suffice for country highway work, but for good leveling the error should not exceed $0.10\sqrt{L}$, or 0.2 in the above circuit, or 0.10 for still better work.

3. In running a line of differential levels, is it essential that the distance from the instrument at each set-up of the +S. and the −S. should be the same, or is it sufficient that the sum of the distances to all the +S. equals that for all the −S.? Ans. The latter is all that is necessary. Intermediate points would be liable to error but the end points would not.

MINOR LEVELING INSTRUMENTS

Locke's Hand Level. *Construction and Operation.* This instrument consists of a brass tube 6 inches long with a small level mounted on its top at one side of the center near the object end, Fig. 52. Underneath the level is an aperture, across which is stretched a horizontal wire attached to a frame. This frame is made adjustable by two opposing screws placed at the ends of the level-tube mounting. In the tube, directly below the level and at 45 degrees to the line of sight, is placed a totally reflecting prism acting as a mirror. The images of the bubble and the wire thus are reflected to the eye. The prism divides the section of the tube into halves, in one of which is seen the bubble and the wire focused sharply by a convex lens placed in the draw tube at the eye end of the instrument, while the other permits an open view. By putting the instrument to the eye and raising and lowering the object end until the bubble is bisected by the horizontal wire, natural objects in the field of view can be seen through the open half at the same time, and approximate levels can then be taken. To prevent dust and dampness from entering the main tube, both the object and the eye ends are closed with plain glass.

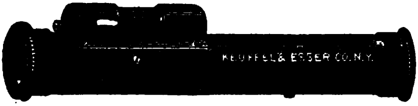


Fig. 52. Locke's Hand Level

The instrument is intended to be carried in the pocket and is of especial value upon reconnoissance surveys where it can be used to get a rough idea of elevations and levels. It is also used in taking cross-sections in railroad and similar work, as explained in "Plane Surveying", Part III.

Adjustment. It is seldom necessary to adjust this instrument, but it should be tested and adjusted, if found necessary, in the following manner: Establish a horizontal line on the side of a building by means of a wye or a dumpy level and mark each end by a nail. Place the hand level on the nail at one end and note whether the bubble is bisected by the wire when the latter is sighted on the farther nail. If not, adjust by means of the screws at each end of the bubble tube. These screws move the wire, and one should be loosened before the other is tightened.

Abney Hand Level and Clinometer. This instrument, Fig. 53, is similar to the Locke hand level, but the small spirit level mounted on top can be moved in the vertical plane and is clamped to a dial graduated upon one side into single degrees and upon the other into slope ratios, so that it is possible to measure angles of slope. The adjustments of the instrument are similar to those for the Locke hand level.

The instrument can be used in the field in the same manner as the Locke hand level but is of more universal application. It is of especial value upon steep slopes when the efficiency of the Locke level would be limited by the length of the rod. In using the Abney instrument, it is necessary only to mark the height of the eyes upon the rod. In sighting upon the rod, with the horizontal line coinciding with the mark upon the rod, move the vertical circle until the bubble is in the center of the tube. Read the vertical angle. The tangent of this angle multiplied by the horizontal distance to the rod will give the difference in elevation. If the distance to the rod is measured along the slope of the ground, multiply this distance by the sine of the vertical angle to get the difference in elevation.

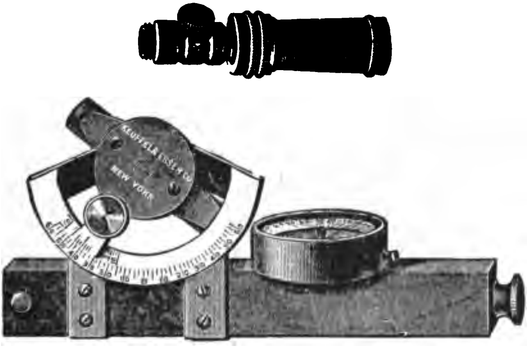


Fig. 53. Abney Hand Level and Clinometer

TRIANGULATION STATION ON MOUNT UNCOMPAHGRE. ELEVATION 14,300 FEET

Courtesy of U. S. Coast and Geodetic Survey, Washington, D. C.

PLANE SURVEYING

PART II

COMPASS AND TRANSIT MEASUREMENTS

Instruments for Measurement of Angles. For measuring angles in surveying work two instruments are in common use, the compass and transit. The compass is a very old instrument and formerly was used almost entirely for land surveys. In recent years, owing to the increase in the value of land which has required more accurate work than is possible with the compass, it has been largely superseded by the transit. However, the compass is still a very valuable instrument and is, without doubt, the better to use in surveys in wooded country where great accuracy is not required. Almost every transit has a compass attached to it, which is used in determining an approximate north line and in checking traverses.

SURVEYOR'S COMPASS

Characteristics of Compass. The surveyor's compass consists primarily of a nonmagnetic circular brass box carrying on a pivot in its center a magnetized steel needle. This needle, being free to revolve about the pivot, will swing back and forth and finally come to rest pointing in the direction of the magnetic pole of the earth. This property of the compass needle became known to Europeans about the twelfth century, but the Chinese are supposed to have known of it about 2634 years B.C. It will be noted also that a compass needle points in the direction of the magnetic poles of the earth; in fact, it places itself in the direction of the magnetic lines of force of the earth. Unfortunately, the magnetic poles, of which there are two—one north and one south—do not coincide with the geographic poles, and furthermore the magnetic poles do not stay in the same place but are constantly changing, owing to causes which are still unknown.

Variation of Magnetic Declination. *Difference between True and Magnetic Meridians.* The true meridian plane of any place upon the surface of the earth is a vertical plane passing through the poles of the earth, while a true meridian, or north and south line, is a line lying in this plane. The magnetic meridian is defined as the direction of the line indicated by the position of the magnetic needle. At a few places upon the surface of the earth, the true meridian and the magnetic meridian coincide at times only, but as noted above, they differ, for the most part, by an ever-varying quantity. The angle at any place between the true meridian and the meridian as defined by the magnetic needle is called the magnetic declination for that place. If the direction of the magnetic meridian were constant or if the changes followed any particular law, it would be a comparatively simple matter to determine the declination for any time or place. The variations occurring, however, are of three principal kinds—diurnal, annual, and secular—the last being the most important.

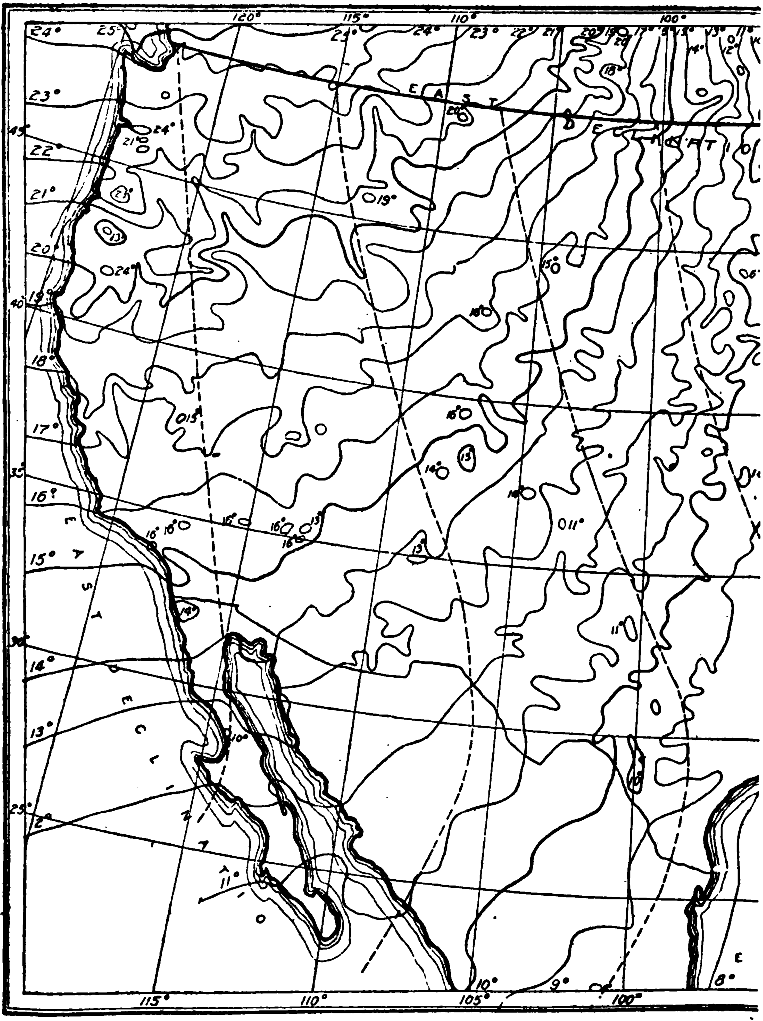
Diurnal Variation. On continuing observations of the direction of the needle, it will be found that the magnetic needle does not point in the same direction but varies slightly throughout the day.

Annual Variation. If observations are continued throughout the year, it will be found that the diurnal changes vary with the seasons, being greater in summer than in winter.

Secular Variation. If accurate observations on the declination of the needle, in the same place, are continued over a number of years, it will be found that there is a continual increase or decrease of the declination from year to year.

Irregular Variations. Besides the three variations just mentioned, the declination is subject to variations more or less irregular, owing to local conditions, lunar perturbations, sun spots, magnetic storms, etc. Local attraction may be expected in the vicinity of industrial or electric plants, steel structures, railroads, iron-ore deposits, electric or other pole lines, etc.

Allowance for Variation. Excepting the secular, these variations are generally unimportant to the surveyor, but that one is of great importance because many of the old land surveys were made with a compass and by using the magnetic north and south. In going over such surveys today, it is necessary to find out how much difference there is between the pointing of the compass today



PLAT

Reproduced from Chart of U. S. Coast and Geodetic Survey. The Lines of Equal Magnetic Declination
Pointed True North and South. The Lines of Equa

and its pointing when the survey was made. This may be found for any part of the United States by consulting the charts issued from time to time by the United States Coast and Geodetic Survey; the chart for 1910 is reproduced (see Plate I). On this chart all points at which the needle points to the true north are connected by lines called *agonic lines*, or *lines of no declination*. Lines called *isogonic lines* are also drawn, connecting points of the same declination. These lines do not show the direction in which the compass needle points but are simply lines of equal magnetic declination for 1910. At all points on the agonic line the needle pointed north at this time, and the declination at other places may be found from the chart, a plus (+) sign indicating that the needle pointed west of true north, and a minus (−) sign that it pointed east. Lines of annual change are also shown.

ILLUSTRATIVE EXAMPLE

Find the declination at New York City, April, 1916. From the chart, the declination in January, 1910, was about 9 degrees 30 minutes, and the annual change is about $5\frac{1}{3}$ minutes, hence the declination would be $9° 30' + (5\frac{1}{3} \times 6\frac{1}{3})$, or 10° 4′. This result is approximate but is close enough for many purposes, as it is generally impossible to get a direction closer than 5 or 10 minutes with a compass.

In making a survey at the present time, the declination should be determined by observation, as described later, and only true directions, not magnetic, should be shown on the map.

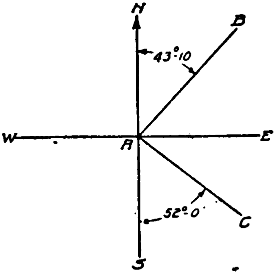


Fig. 54. Diagram Showing Bearings of Lines

Bearings. Angles are not read directly by means of the compass but may easily be computed from the compass readings or bearings. The bearing of a line is the angle it makes with the north and south line measured east or west from this line. Therefore a bearing never exceeds 90 degrees in value, and we must always state what quadrant the bearing is in. Thus, in Fig. 54, the bearing of the line *AB* is N 43° 10′ E and that of the line *AC* is S 52° 0′ E. The angle at *A* between *B* and *C* is easily found to be 180° − (43° 10′ + 52° 0′), or 84° 50′.

COMPASS CONSTRUCTION AND OPERATION

Compass Box. The surveyor's compass, Fig. 55, as has been noted, consists primarily of a circular brass box carrying on a pivot in its center a magnetized needle. The inside edge of the box, on a level with the needle, is usually graduated to half degrees, and smaller intervals may be estimated. Two points diametrically opposite each other are marked 0°, Fig. 56, and form the north and south ends of the box, the south end being indicated by the letter S and the north end either by the letter N or by a fleur-de-lis, as in Fig. 55, or other striking figure. The scale divisions are numbered up to 90 degrees on both sides of these points to the east and west points marked E and W, respectively.

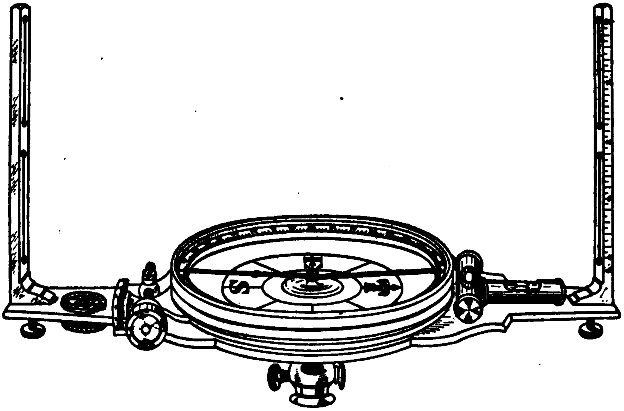


Fig. 55. Surveyor's Compass

Bearing Indication. To read a bearing, the compass is set up over a point, the north end turned so as to be ahead, or in advance, of the position of the observer, and the sights are turned until they point along the line; the observer then reads the pointing of the north end of the needle on the scale. When reading the needle, the observer should keep directly behind the needle so as to avoid reading from one side or the other. Thus, in Fig. 56, a line through the sights *aa* is northeast and is found by reading the north end of the needle, being about N 30° 30′ E. It is noticed that the E and W marks are reversed from their usual positions; this is necessary in order that the reading on the scale shall indicate the quadrant in which the sights are pointing.

Magnetic Needle. The magnetic needle is the most essential part of the compass. It consists of a slender bar or tube of steel, usually 5 or 6 inches long, strongly magnetized and balanced on a pivot so that it may turn freely and thus continue to point in the same direction however much the box carrying the pivot may be turned around. To facilitate this, the pivot should be of the hardest steel ground to a very fine point, and the center of the needle resting on the pivot should be fitted with a cap of agate or other hard substance.

To distinguish the ends of the needle, the north end may have a special shape or carry an ornamental point; but usually it is plain and differs from the other end only in that the south end carries a coil of wire, as shown in Figs. 55 and 56, to balance the *dip* or tendency of the north end of the needle (for locations north of the equator) to point toward the magnetic north pole.

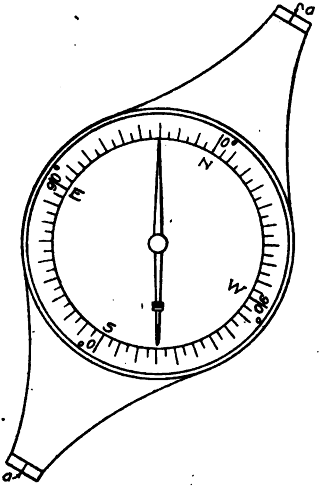


Fig. 56. Plan View of Surveyor's Compass

Intensity of directive force and sensitiveness are the chief requisites in a magnetic needle, and nothing is gained by making a needle more than 5 inches in length. The needle should not come to rest too quickly. Its sensitiveness is indicated by the number of vibrations that it makes in a small space of time before coming to rest. Should it come to rest quickly or be sluggish in movement, it indicates that either the magnetization is weak or there is undue friction between the needle and pivot.

The underside of the box should be fitted with a lifting screw which, engaging a lever on the inside of the box, will serve to raise the needle off the pivot when the instrument is carried about and will thus prevent wear on the point.

Sights. The sights form the next most important feature of the compass. They usually consist of two brass uprights with a narrow slit in each; or the upright at the north end may have a wide opening and carry a vertical wire. These sights are mounted directly on the compass box, or the bottom of the box may be extended at each end in the form of a plate and the sights attached at the ends of the plates. However mounted, the sights should have their slits in line directly over the north and south points of the divided circle.

Reading Vertical Angles. The rear sight may have peepholes, as shown in Fig. 55, and both the right and left edges of the upright at the north end may be divided by a series of graduations to enable the observer to determine vertical angles roughly. To read small angles of depression, the observer looks through the upper circular aperture, or peephole, in the rear upright and past the forward upright to the point of lower elevation. The point at which the line of sight passes the left side of the forward upright is marked by a card or pencil, and the vertical angle is read from the scale. Angles of elevation are determined in a similar manner, except that the lower peephole of the rear upright and the scale on the right side of the forward upright are used. The scale on the right side of the forward upright increases from the bottom up, while the left scale increases from the top down.

Leveling Devices. *Level Bubbles.* Two level bubbles for leveling the box are placed at right angles to each other in the bottom of the compass box or mounted in the same way on the plate.

Ball-and-Socket Joint. The compass is usually fitted to a spindle made slightly conical, which has on its lower end a ball turned perfectly spherical confined in a socket by a pressure so light that the ball can be moved in any direction in leveling the instrument. The ball is placed in the brass head of a Jacob's staff or, better, in the top casting of a tripod.

Plumb Bob. A plumb bob should be provided with the instrument to center it over a stake.

Prismatic Compass. The prismatic compass is a hand compass for very rough work. It is about 3 inches in diameter, with a floating metal dial, and is provided with folding sights and prism. By means of the prism it may be read while being pointed. This is especially useful when the instrument is held in the hand. Although it can be mounted on a Jacob's staff, the compass is usually held in the hand and, when not in use, is carried in the observer's pocket.

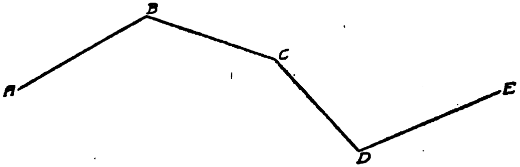


Fig. 57. Open Traverse Survey

Compass Adjustments. The compass is not a highly accurate instrument and the adjustments are usually made by the maker in a permanent manner. They should be tested from time to time, however, and adjusted, if necessary, in the following manner:

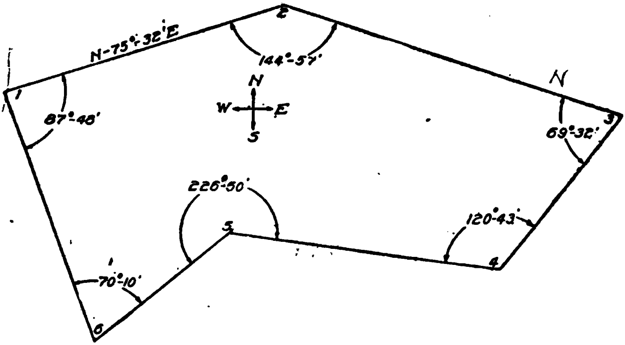


Fig. 58. Closed Traverse Survey

Adjustment of Plate Levels. The axes of the bubble tubes attached to the plate should be parallel to the plate, so that the plate and box will be horizontal when the bubbles are in the middle of the tubes. Actually, the vertical axis about which the instrument turns is made by the maker at right angles to the plate, and the adjustment consists in making the axes of the bubble tubes also at right angles to the vertical axis and hence parallel to the plate. This is done by the method of reversion as follows:

First, bring the bubbles to the middle of the tubes by the pressure of the hand on the plate, which can be raised or lowered in any direction by reason of the ball-and-socket joint; then revolve the plate 180 degrees, so that the tubes will be reversed end for end. If the bubbles remain in the middle of the tubes, the tubes are in adjustment, that is, are parallel to the plate. If the bubbles do not remain in the middle, raise or lower one end of each tube by tightening or loosening the screws for that purpose at the end of the tubes until the bubble is brought back one-half the distance. Bring the bubbles to the middle of the tubes by releveling, and repeat the test and correction until the bubbles remain in the middle with a complete revolution of the plate. Note that on many compasses no adjusting screws are provided except those to fasten the tubes to the plate. The adjustment is made by loosening the screws and placing small pieces of paper under the low end of each tube.

Adjustment of Needle to Dip. While the compass is still in a perfectly level condition, see if the needle is in a horizontal plane. Should this not be the case, move the small coil of wire toward the high end until the needle swings horizontally.

Adjustment of Compass Sights. The sights should be vertical when the plate is level. Level the plate and sight on the string of a suspended plumb bob. If the slits in the upright are parallel to the string, they are vertical; if not, they must be made vertical by removing them and filing the bottoms or by inserting pieces of paper under the edges. The sights should also be tested to see that the line of sight passes through the zeros on the circle. This can be done by stretching a thread between the slits and observing whether the N and S marks are in the plane of the threads. If badly out, correction should be made by the maker.

Adjustment of Compass Needle. The needle should be straight and the pivot should be at the center of the graduated circle.

Read both ends of the needle, preferably, when one end is set on an even graduation. If the needle is straight and the pivot is in the center of the graduated circle, the difference in readings will be 180 degrees. If the difference is other than 180 degrees, it may be owing to a bent needle, to the pivot being off center, or to both. Obtain readings of both ends for a number of positions of the plate. If the difference between the end readings is constant, the error is due to a bent needle and the pivot is in the center. To correct, remove the glass on the box and bend the needle straight. If the difference in readings is not constant, the error may be due to the pivot being off center or to a bent needle combined with the pivot being off center. To correct, bend the pivot in a direction at right angles to the position of the needle giving the greatest difference in end readings. Repeat the test and, if necessary, bend the needle. The operations of testing and correcting should be repeated until the ends of the needle are 180 degrees apart for all parts of the circle.

METHODS OF USING COMPASS

Essentials of Process. In the process of locating points, and therefore lines, by angle-measuring instruments, two operations are necessary, as follows: measuring the angle at the instrument between some given line and the line passing through the given point; and measuring the distance from the instrument to the given point. The bearing and length of a line are collectively named the *course*. The compass, as already described, measures bearings directly and angles indirectly.

Determination of Bearing. To determine the bearing of a line proceed as follows: Set up the compass over one point and level carefully. Turn the sights in the approximate direction of the second point, with the north end of the plate ahead. Hold a rod on the second point and carefully cover it with the slits in the sights. Now lower the needle on the pivot, being sure that the instrument is still level, allow it to come to rest, and read the bearing.

Survey of Series of Lines. To survey a series of lines with the compass, the process is as follows: Set up the compass over the point A, Fig. 57, with the north end of the plate ahead and, after leveling, turn the sights to cover a rod held on the point *B*. Send out the tape in the direction of *B* and, sighting through the slits, signal the head tapeman into line and measure the distance to point *B*. Read and record the bearing together with the length of the line. Take up the instrument, carry it to *B*, and set it up over that point. Level, turn the north end toward *A*, and sight on the rod held on *A*. Read the bearing as a check on the former one, but reversed in direction, that is, if the bearing from *A* to *B* is north by east the bearing from *B* to *A* will be south by west. If the direct and reversed bearings check within a reasonable amount—ordinarily 10 to 15 minutes—turn the north end of the compass to cover a rod held on *C*. Read and record the bearing, measure *BC* and record, then take the instrument to *C* and proceed as before. This process is known as *traversing.*

If, at any station, such as *C*, the forward and back bearings do not agree, return the instrument to *B* and again take the bearing of *BC*. If they still disagree it indicates local attraction at *C*, that is, there is something, such as an iron fence, iron deposit, or other magnetic attraction which causes the needle to be pulled away from its correct position. Take the instrument to *D* and take the bearing of *DC*, comparing it with the bearing of *CD*. If these disagree, record the bearings of *BC* and *DC* as well as those of *CB* and *CD*. The angle *BCD* can be computed from both sets of bearings and should check, since both bearings read at *C* will be affected an equal amount by local attraction, and hence their difference will give the true angle between *BC* and *CD*. Should it be found that the readings at a number of consecutive stations are affected by local attraction, the angles between the lines at each station must be determined and the bearings computed.

Locating a series of lines with certain lengths and bearings is essentially the same process as the above, except that after the compass has been turned in the proper direction the stations must be brought into proper line.

Open and Closed Traverses. A series of lines like that shown in Fig. 57 is an open traverse. If the traverse continued and finally came back to *A*, forming a closed figure, as in Fig. 58, it would be a closed traverse.

Precautions in Use of Compass. Sometimes, as when the line of which the bearing is required consists of a fence or other obstructions, the compass cannot be set on the line. In such a case measure off equal distances at right angles to the line and find the bearing of the parallel line; the length should be measured on the line itself. In other cases it may be more convenient to set the compass or rod in line on the line produced or on some intermediate point of the line.

Avoid reading the bearing from the wrong one of the two numbers between which the needle points, as, for instance, 35 degrees for 25 degrees.

Stop the vibrations of the needle by gently raising it off the pivot and lowering it again by means of the screw on the underside of the box.

If the needle is slow in starting, smartly tap the compass to destroy the effect of any possible adhesion to the pivot or friction of dust upon it.

Avoid holding the pins, ax, or any other body of iron in close proximity to the needle.

Should the needle adhere to the glass after the latter has been dusted with a handkerchief or has been carried so as to rub against the clothes, the trouble is caused by the glass being thereby charged with electricity and may be obviated by moistening the finger and applying it to the glass.

The north-end sight should always be pointed forward when sighting at a point, and the north end of the needle should be read for the bearing.

The needle should be raised from the pivot when not in use.

Error of Closure. If it were possible to plot the field notes of a closed traverse exactly, starting from the first point and laying off each bearing and distance, it would be found that when the last distance and bearing were laid off, the point so located would not coincide exactly with the starting point. There would be an error of closure that would represent the aggregate effect of all the errors made in measuring the bearings and distances in the traverse. The amount by which the survey fails to close, or the error of closure, is a measure of the accuracy of the field work, since careful measurement of bearings and lengths will result in small errors of closure, while hasty and incorrect measurements will give large closing errors.

Practical Limits. The error of closure for surveys made with a compass and tape can usually be kept within 1 in 600 or better, that is, 1 foot for each 600 feet of perimeter of survey. The amount by which the survey fails to close can be determined exactly by the method of latitude and departures, as explained later.

It should also be noted in this connection that if the radius is 300 feet, 1 minute of arc equals a chord of 0.1 foot. The relation between angular and linear errors, therefore, is such that if the angles are read to the nearest minute, distances should be chained so as to obtain an accuracy of about 1 in 3000. Angles obtained by a compass will usually be subject to an error of at least 5 minutes, hence there is no necessity for trying to do the chaining for compass surveys more accurately than 1 in 600 or, say, 1000, so as to insure the distances being at least as accurate as the angles.

Methods of Recording Field Notes. Field notes may be recorded in various ways—two of which are as follows—the object being to make them clear and full:

(1) The surveyor may make a rough sketch of the survey by eye in the field book and note on the lines their bearings and lengths.

(2) He may write the stations, bearings, and distances in three columns on the left-hand page of the notebook, and on the right-hand page draw a rough sketch of the survey, noting on this sketch the approximate relative positions of the lines, with bearings, lengths, etc., marked. Field notes for a compass survey may be tabulated as follows:

Stations	Bearings	Distances (feet)
0	N 32° E	1110.12
1	S 36° E	1247.40
3	S 27½° W	518.10
4	S 16° W	1009.80

Use of Compass in Land Surveying. The detailed use of the compass and methods followed in land surveying will be treated later under Land Surveying, in "Plane Surveying", Part III.

EXAMPLES FOR PRACTICE

1. The magnetic bearings of four courses are found to be N 51° 20′ E, S 88° 5′ W, S 85° 10′ E, and N 3° 5′ W. Suppose the declination is 8° 30′ W. Find the true bearings.

Ans. N 42° 50′ E, S 79° 35′ W, N 86° 20′ E, and N 11° 35′ W.

2. On some compasses, Fig. 55, there is a device by means of which the zero of the scale may be moved to the left or right, the whole scale turning in the box. The amount the scale is moved from the initial position in which it coincides with the line through the sights can be measured on a small vernier placed at the north end of the box. By thus moving the zero of the scale an amount equal to the declination, the compass will read true bearings. If the declination is 10° 20′ W, how should the zero of the scale be moved?

Ans. It should be moved this amount to the left of the sights, so that when the needle reads zero and is pointing magnetic north, the sights will be pointing 10° 20′ to the east of the needle, or true north.

3. Given the closed traverse, shown in Fig. 58, in which all the interior angles have been measured and adjusted, and given also the bearing of the side *1–2*, as N 75° 32′ E, as shown, compute the bearings of the other sides.

Ans. *2–3*, S 69° 25′ E; *3–4*, S 41° 3′ W; *4–5*, N 79° 40′ W; *5–6*, S 53° 30′ W; *6–1*, N 16° 40′ W.

4. In certain kinds of work with the compass, it is convenient to assume one of the lines as a meridian, and it then becomes necessary to change the bearings of all the other lines to conform with the assumed meridian. The bearings of the sides of a field are tabulated below.

Stations	Bearings	Distances (feet)
1	N 35° 0′ E	178.20
2	N 83° 30′ E	85.14
3	S 57° 0′ E	146.52
4	S 34° 15′ W	234.30
5	N 56° 30′ W	213.18

Suppose, now, that the first course is assumed as a meridian, that is, that its bearing is due north and south; what then would be the bearings of the remaining courses?

Ans. Since the courses are changed to the west by 35 degrees, the new bearing for course *2* will be N 48° 30′ E; for course *3* it will be 57°+35°=92°, or the new bearing will be N 88° 0′ E; for course *4*, it will be 34° 15′−35°=0° 45′ in the next quadrant, or the bearing will be S 0° 45′ E; for course *5*, it will be 56° 30′+35°=91° 30′, or the bearing will be S 88° 30′ W.

5. Referring to Fig. 57, note the tabulated forward and back bearings which were measured in the field. They indicate local attraction; what are the best values of these bearings to use?

Course	Bearings	
	Forward	Back
A–B	N 61° 10′ E	S 61° 20′ W
B–C	S 70° 40′ E	N 68° 30′ W
C–D	S 37° 10′ E	N 35° 10′ W
D–E	N 72° 20′ E	S 72° 10′ W

Ans. Since the forward and back bearings agree as closely as can be expected for compass work at stations *A* and *D*, use the mean values for courses *A–B* and *D–E*, or N 61° 15′ W and N 72° 15′ E. Also, since the back bearing of *A–B* agrees with the forward bearing, there can be no local attraction at *A* or *B*, nor, for a similar reason, at *D* or *E*. Therefore the local attraction must be at *C*, and, there being none at *B*, the forward bearing *B–C*, or S 70° 40′ E, must be substantially correct. This means that the back bearing, N 68° 30′ W, is too low by 2 degrees 10 minutes and indicates that there is some influence near *C* that pulls the compass needle about this amount to the west of its true position. If this is so, the forward bearing *C–D* should be too large by about 2 degrees 10 minutes or should be S 35° 0′ E, checking within 10 minutes with the back bearing of this course, which we know is about right, as it was taken from *D*. Using the average value of the correction, or 2 degrees 5 minutes, we get the corrected bearings of *B–C* and *C–D*, which are S 70° 35′ E and S 35° 5′ W.

COMPUTATIONS IN COMPASS SURVEYS

Method of Latitudes and Departures. If a survey has been made and the bearings and lengths of the sides determined, it is then necessary to test the survey, that is, to find the error of closure and, if this is satisfactory, to proportion in a reasonable manner the error which occurs, so as to obtain values that agree or check, and finally to obtain the area. This is done by the method of latitudes and departures.

Latitude and Longitude Differences. The latitude of a point is its distance north or south of some line running east and west taken as a parallel of latitude.

The longitude of a point is its distance east or west of some line running north and south taken as a meridian.

The distance that one end of a line is north or south of the other end is the difference of latitude of the two ends of the line and is called its *northing* or *southing* or simply its latitude.

The distance that one end of a line is east or west of the other end is the difference of longitude of the two ends of the line and is called its *easting* or *westing* or its departure.

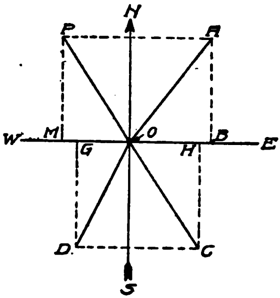


Fig. 59. Illustrating Latitude and Longitude Differences

The terms latitude difference and longitude difference have of late come into quite general favor and will be adhered to in what follows.

Trigonometrical Relations. In Fig. 59, SN represents a meridian and WE a parallel of latitude. If we take the line OA, its bearing as given by the compass is the angle NOA. The latitude difference, or northing, of the line OA, therefore, is AB, which equals $OA \cos NOA$. Its longitude difference, or departure, or easting, is OB, which equals $OA \sin NOA$. Hence, to find the latitude difference of a course, *multiply the length of the course by the cosine of the bearing;* and to find the longitude difference of any course, *multiply the length of the course by the sine of the bearing.*

It is also the custom to designate the direction of a course by the convention used in Trigonometry, that is, if the course is NE or NW, the latitude difference will be north and will be designated by the plus (+) sign; if the course is SE or SW, the latitude difference will be south and will be designated by the minus (−) sign. If the course is NE or SE, the longitude difference will be east and will be designated by the plus (+) sign; if the course is NW or SW, the longitude difference will be west and will be designated by the minus (−) sign.

Thus, in Fig. 59, OA is of plus latitude difference and plus longitude difference, OP is plus and minus, OD is minus and minus, and OC is of minus latitude difference and plus longitude difference.

Use of Logarithmic Tables. For calculating latitude and longitude differences a table of natural functions can be used, but much time and labor can be saved by using logarithms, so the student should procure a table of logarithms and become familiar with their use. For all-around use Bruhns' or Véga's 7-place tables are best. A student should remember that it is useless to carry computations to any greater number of decimal places than is warranted by the accuracy of the field measurements. If a distance is measured to 10ths of feet only, and this is liable to be out by 0.1 or more, it is a waste of time and money to carry the computations any closer than 10ths. In general, this would require four accurate figures and 5-place logarithms could be used. The 7-place tables, however, save time in interpolating, even where only the 5-place are required, and will be good enough for the most accurate work, hence, are recommended. In offices doing much surveying work, some form of traverse table is advisable, and a very satisfactory arrangement is the use of Gurden's Tables (Van Nostrand, New York) in connection with an adding machine.

Test of Survey by Latitude and Longitude Differences. It is evident that after the surveyor has gone completely around a field or farm, measuring all the lengths and bearings, returning to the starting point, he has gone as far north as south and as far east as west. In other words, if the work has been done correctly, the algebraic sum of the latitudes must equal zero and the algebraic sum of the departures must equal zero. This condition, however, will seldom be attained, and it becomes necessary to determine the error and note whether it is satisfactory. This process can be tested very easily by calculating the latitude and longitude differences of the survey analyzed on page 91 and illustrated by the diagram, Fig. 68. The data for a typical survey is given below arranged in nine columns.

Course	Bearing	Cosine	Sine	Distance (ft.)	Latitude Difference		Longitude Difference	
					N (ft.)	S (ft.)	E (ft.)	W (ft.)
1-2	S 21° 0′ W	0.934	0.358	124.1	.....	115.91	.....	44.43
2-3	N 83° 15′ E	0.118	0.993	58.6	6.91	......	58.19	.....
3-4	N 12° 0′ E	0.978	0.208	82.5	80.69	......	17.16	.....
4-1	N 47° 0′ W	0.682	0.732	42.4	28.92	......	.....	31.04
Total	....	.....	.....	307.6	116.52	115.91	75.35	75.47

Error of Closure. The plus latitude differences fail to equal the minus latitude differences by 0.61 foot, and the plus longitude differences fail to equal the minus longitude differences by 0.12 foot. These values are the latitude and longitude differences of the error of closure, which is the length of a course required to make the algebraic sum of the latitude differences and the longitude differences equal. The error of closure, therefore, in this case, is $\sqrt{(0.61)^2+(0.12)^2}$, or 0.62 foot. The perimeter, or total length of traverse, is 307.6 feet. Hence an error of 0.62 foot is in the proportion of about 1 in 500 which is not very satisfactory even for a compass traverse but might be passed in rough work. If this is not satisfactory, some idea of where the error occurred may be obtained by examining the above data. It will be noted that the error is mainly in the latitude differences. This would indicate that if the angles are reasonably accurate—and this can be tested by computing the interior angles and noting how near their sum checks with 180°(n-2), n equaling the number of sides—the error is probably in a line that runs almost north and south We would therefore go into the field and remeasure *1-2* and *3-4* and probably find the error in these courses, thus saving the time and labor of repeating the entire survey. A similar analysis may be made for other conditions.

Interior Angles. In checking the sum of the interior angles, it will usually fail to exactly equal 180°(n–2). For good work with a compass the error in minutes should be about $5'\sqrt{n}$, in which n is the number of sides. In the above example the angles and bearings have been adjusted to exactly equal 360 degrees by distributing the error equally, but this is seldom done.

Balancing of Survey. Before proceeding to the calculation of the content of a field or farm, the survey must be balanced, that is, the latitudes and departures must be corrected so that their sums shall be equal or balance. Whether the bearings or lengths shall be corrected will depend somewhat on the conditions under wnich the survey was made. If the surveyor has reason to think that the error is entirely in the bearing of one or more or even of all the courses the corrections may be made accordingly. If, on the other hand, one or more of the courses were measured over difficult ground, it may be presumed that the error occurred in those lines. If, however, there is no reason to believe that one course is in error more than another, the differences may be distributed among the courses in proportion to their lengths according to the following proportions:

As the length of any course is to the sum of the lengths of all the courses so is the correction of the latitude difference of that course to the total error in the latitude differences of all the courses.

As the length of any course is to the sum of the lengths of all the courses so is the correction of the longtitude difference of that course to the total error in the longitude differences of all the courses.

The practical application of these proportions to balancing a survey may be illustrated from the example, page 81 (see also Fig. 68). Since the sum of the plus latitude differences exceeds that of the minus latitude differences, these corrections must be subtracted from the plus differences and added to the minus differences, that is, the balanced latitude difference for course *1–2* will be 115.91 plus 0.25, which equals 116.16, and similarly for the other courses, giving the values shown in the tabulation herewith, and exactly balancing. The corrections for longitude differences are subtracted in this case from the minus and added to the plus longitude differences, the final balanced results being as indicated.

Course	Correction x	Latitude Difference		Correction y	Longitude Difference	
		N+	S−		E+	S−
1-2	0 25	.....	116.16	0.05	.....	44.38
2-3	0.12	6.79	......	0 02	58.21	.....
3-4	0.16	80 53	......	0.03	17.19	....
4-1	0.08	28 84	....	0.02	.....	31 02
Total	....	116.16	116.16	....	75 40	75.40

Calculation of Content. After a field has been surveyed, its content may be calculated by dividing it into triangles, trapezoids, etc., calculating their various contents and adding them together. This, however, is at best a cumbersome method involving much work of calculation and great chance of error. The method of latitude and longitude differences is at once simple, easily applied, and easily checked. Before proceeding to develop a formula for this method, it will be necessary to illustrate and define certain terms.

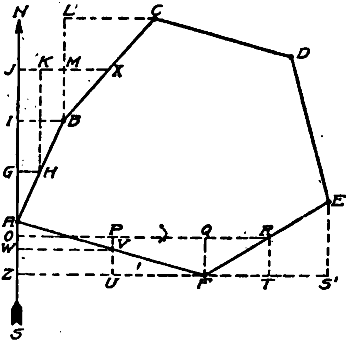


Fig. 60. Illustrating Latitude and Longitude Differences and Double Longitude

Draw a line, as *SN*, Fig. 60, through the extreme east or west corner of the field for a meridian. From the definitions previously given, the longitude difference of *AB* is *IB;* that of the line *BC* is *LC;* that of *EF* is *S'F;* and that of *AF* is *OQ*.

The perpendicular distance of each station from the given meridian is the longitude of that station—plus, if east; minus, if west. Thus the longitude of *A* is zero; that of *B* is *IB;* that of *C* is *IB*+*LC;* that of *E* is *OQ*+*FS';* and that of *F* is *OQ* which equals *ZS'*−*FS'*.

Similarly, the latitude difference of *AB* is *AI;* that of *BC* is *BL;* and that of *EF* is *ES'*.

The distance of the middle of any side of a field from the meridian is called the longitude of that side. Thus the longitude of the side *AB* is *GH;* that of *BC* is *JX* which equals *GH*+*KM*+*MX;*

and that of AF is WV which equals $OR-QR-QP$, the minus signs being used in this instance because the lines EF and AF bear to the west. An analysis of WV will show that it equals $OR-RQ-QP$, OR being the longitude of the preceding course, RQ being one-half the longitude difference of the preceding course, and QP being one-half the longitude difference of the course itself.

To avoid fractional quantities, it is customary to double the preceding expressions and to call the result the double longitude.

The double longitude of the first course equals the longitude difference of that course. The double longitude of the second course equals the double longitude of the first course plus the longitude difference of the first course plus the longitude difference of the second course.

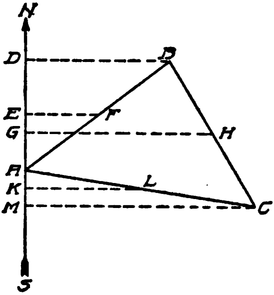


Fig. 61. Area by Double Longitudes

The double longitude of any course equals the double longitude of the preceding course plus the longitude difference of the preceding course plus the longitude difference of the course itself.

Area of Three-Sided Field. We shall now proceed to deduce a rule for determining areas by double longitudes and shall first take a 3-sided field, Fig. 61.

Drawing a line through the most westerly corner A, we see that the area of the field will be the difference between the area of the trapezoid $DBCM$ and the combined area of the triangles DBA and ACM. The double area of the triangle DBA is the product of DB by DA, or the double longitude of AB by the latitude difference of AB. The resulting product will be north, or plus. The double area of the trapezoid $DBCM$ is the product of $DB+MC(=2GH)$ by DM, that is, the double longitude of BC by its latitude difference. The resulting product will be south, or minus. The double area of the triangle ACM will be the product of MC by AM, or the double longitude of the course AC by its latitude difference. The resulting product will be north, or plus. Adding together, then, the plus products and subtracting from the minus product gives as the result the double area of the field.

Area of Four-Sided Field. We shall next take a 4-sided field, Fig. 62.

It is evident that the area of the field $ABCD$ is the difference between the sum of the areas of the two trapezoids $TBCR$ and $RCDE$ and the sum of the areas of the triangles ABT and ADE. The double area of the triangle ABT is the product of BT by AT, or the product of the double longitude of the course AB by its latitude difference. The result will be a north product, or plus. The double area of the trapezoid $TBCR$ will be the product of $TB+CR(=2LP)$ by TR, that is, the product of the double longitude of the course BC by its latitude difference. The result will be a south product, or minus. The double area of the trapezoid $RCDE$ will be the product of $RC+DE$ $(=2FK)$ by RE, or the product of the double longitude of the course CD by its latitude difference. The result will be a south product, or minus. The double area of the triangle ADE will be the product of ED by AE, or the product of the double longitude of the course AD by its latitude difference. The result will be a north product, or plus. Finally, adding together the north products, then adding together the south products and taking the difference of their sums gives as the result the double area of the field $ABCD$.

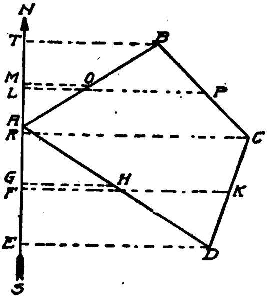


Fig. 62. Area by Double Longitudes

Any Enclosed Area. The same principle will apply to any enclosed area, however great the number of the sides. *The area will always be one-half the difference of the sums of the north and the south products arising from multiplying the double longitude of each course by its latitude difference.*

Systematic Computation of Content. For systematic computation, arrange the work as follows:

(1) Arrange the columns as in the example, page 81.

(2) Balance the latitudes and departures, putting the corrected quantities above the others in red ink; or else arrange four additional columns and enter them in their proper places.

(3) Compute the double longitude of each course with reference to a meridian passing through the extreme east or west station and place the results in another column.

(4) Multiply the double longitude of each course by the corrected latitude of that course and place the north products in one column and the south products in another.

(5) Add together the north products and also the south products and take the difference of their sums. Divide the difference by 2, and the result will be the area desired, expressed in the unit of the survey.

(6) If the survey has been made with a Gunter chain, the result will be in square chains. Divide by 10 to reduce to acres.

(7) If the survey has been made with a tape graduated in feet, the result will be in square feet; to express this in acres it may be divided by 43,560.

ILLUSTRATIVE EXAMPLE

We shall now proceed to calculate the content of the field given by the notes on page 81 and illustrated in Fig. 68.

The arrangement of the columns for convenient calculation is as described under Systematic Computation of Content. Upon making a rough sketch of the courses, it is found that station *3* is the farthest east, and therefore the double longitudes will be calculated beginning with course *3*. From the definition previously given, the double longitude of course *3* is equal to its longitude difference, or +17.19. The double longitude of course *4* equals the double longitude of course *3* plus the longitude difference of course *3* plus the longitude difference of course *4*, or 17.19+17.19 +(−31.02)=+3.36. The double longitude of course *1* equals the double longitude of course *4* plus the longitude difference of course *4* plus the longitude difference of course *1*, or 3.36+(−31.02)+ (−44.38) = −72.04. The double longitude of course *2* equals the double longitude of course *1* plus the longitude difference of course *1* plus the longitude difference of course *2*, or +(−72.04)+ (−44.38)+58.21 = −58.21, which checks with the value of the longitude difference and proves that the double longitudes are correctly calculated. Multiplying these double longitudes by their respective latitude differences gives the quantities in the least two columns—the first, third, and fourth being positive and the second negative. Taking the difference of the sums of the quantities in these columns and dividing the result by 2 gives the content of the field, 4727.1 square feet. Dividing by 43,560 gives 0.1085 acre.

Course	Bearing	Distance (ft.)	Balanced Latitude Difference		Balanced Longitude Difference		Double Longitudes	Double Areas	
			N+	S−	E+	W−		+ Products	− Products
1–2	S 21° 0′ W	124.1	......	116.16	......	44.38	−72.04	8368.2	......
2–3	N 83° 15′ E	58.6	6.79	......	58.21	......	−58.21	......	395.2
3–4	N 12° 0′ E	82.5	80.53	......	17.19	......	+17.19	1384.3	......
4–1	N 47° 0′ W	42.4	28.84	......	......	31.02	+ 3.36	86.9	......

+9849.4 395.2
−395.2
2 | 9454.2
4727.1 sq. ft.,
or 0.1085 acre

Examples*. Calculate the content of the fields from the data tabulated below. The result, where found in square meters, should be reduced to acres; 1 square meter equals 0.000247 acre.

1.

Stations	Bearings	Distance (feet)
1	N 34° 15′ E	180.2
2	N 85° 0′ E	84.5
3	S 56° 45′ E	145.2
4	S 34° 15′ W	233.0
5	N 56° 30′ W	211.2

Stations	Bearings	Distances (meters)†
1	S 5° 35′ W	2388.88
2	S 39° 35′ W	1060.27
3	S 50° 25′ E	3078.31
4	S 79° 5′ E	325.00
5	S 53° 50′ E	275.00
6	S 48° 15′ W	200.00
7	N 82° 45′ E	450.00
8	S 87° 40′ E	186.72
9	N	8768.12
10	N 84° 25′ W	1898.54
11	S 5° 35′ W	3530.60
12	N 84° 25′ W	257.50

Ans. 5000 − acres

---

* From *Gillespie's Surveying* (Staley).

†Make computations in meters and then reduce to acres; 4046.883 square meters equal 1 acre.

Method of Supplying Omissions. The method of latitude and longitude differences may be applied to supplying any two omissions in the field work of a closed survey—whether of the direction, or of the length, or of both, as illustrated below—although this method should be resorted to only in cases of absolute necessity, since any omission renders the checking of the field work impossible. In the following paragraphs the methods outlined will apply equally well, whether the survey has been made with the transit or with the compass.

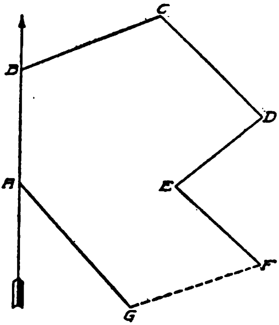


Fig. 63. To Find Length and Bearing of Missing Course

Case 1. Length and Bearing of One Side Wanting. In Fig. 63 let the dotted line FG represent the course whose length and bearing are wanting. Calculate the latitude and longitude differences of the remaining courses; and, since in a closed survey the algebraic sum of the latitude and longitude differences should equal zero, therefore, the amount by which the plus latitude differences fail to equal the minus differences will be the latitude difference of the missing line, and similarly for the longitude differences. Thus, referring to the survey already discussed on page 81 and shown in Fig. 68, and supposing course *3–4* could not be measured, the sum of the other plus and minus latitude differences would be +35.83 and −115.91, respectively. Hence, the latitude difference of the course which would be required to close the traverse would be +80.08. Similarly, the longitude difference would be +58.19 and −75.47. Hence the longitude difference for the closing course is +17.28. The closing course, therefore, must be NE, and these two values are its latitude and longitude differences. The tangent of its bearing equals the longitude difference divided by the latitude difference, or 17.28÷80.08, which gives its bearing; and its length equals the latitude difference divided by the cosine of the bearing. Note that all the error in the survey is thrown into this side and we have no check on the accuracy.

Case 2. Length of One Side and Bearing of Another Wanting. (a) Deficient Sides Adjacent. In Fig. 64 let the bearing of DE and the length of FE be lacking. Draw DF. From the preceding proposition calculate the bearing and length of DF as though DE and EF did not exist. Then, in the triangle DEF, there are given the lengths DF and DE and the angle DFE, from which can be calculated the angle FDE and the length EF.

(b) Deficient Sides Separated. In Fig. 65 let $ABCDEFGA$ represent a 7-sided field, in which the length of CD and the bearing of FG are wanting. Draw DB', $B'A'$, and $A'G'$ of the same lengths and parallel, respectively, to CB, BA, and AG. Connect G' with G, E, and F. Then, in the figure $DB'A'G'E$ there are given the lengths and bearings of all the courses but $G'E$. The length and bearing of the last course can be calculated by the principles of case 1. Then, in the triangle EFG', there are given the lengths and bearings of EF and EG', from which can be calculated the length and bearing of FG'. Therefore, in the triangle GFG', since GG' is equal in length and parallel to CD, there are given the lengths of GF and FG' and the bearings of GG' and FG', from which can be calculated the length of GG' and the bearing of GF, as in case 2 (a).

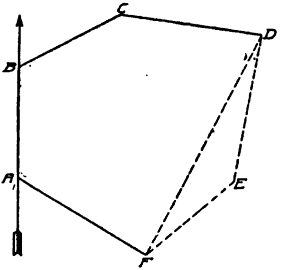


Fig. 64. To Find Length of One Missing Course and Bearing of Another Missing Course

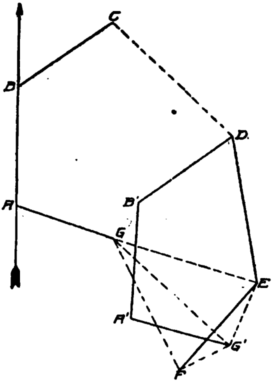


Fig. 65. To Find Length of One Missing Course and Bearing of Another Missing Course When Not Adjacent

Case 3. Lengths of Two Sides Wanting. (a) Deficient Sides Adjacent. In the 7-sided field, Fig. 66, let the lengths of DE and

EF be wanting. Calculate the length and bearing of *DF* by the principles of case 1. Then, in the triangle *EDF*, there are given the angles at *D* and *F* and the length of *DF*, from which can be calculated the lengths of *DE* and *EF*.

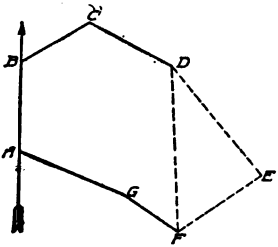


Fig. 66. To Find Bearings of Two Missing Courses

(b) Deficient Sides Separated. In Fig. 65 let the lengths of *CD* and *GF* be wanting. As before, having calculated the length and bearing of *FG'* in the triangle *FGG'*, the angle at *G* can be calculated from the bearings of *FG* and *GG'*, the angle at *G'* from the bearings of *GG'* and *FG'*, and the angle at *F* from the bearings of *FG* and *FG'*. There are given then the three angles of the triangle and the length of one side, from which can be calculated the lengths of the other sides.

Case 4. Bearings of Two Sides Wanting. (a) Deficient Sides Adjacent. In Fig. 66 find the length and bearing of *DF* as before. Then, in the triangle *DEF*, there are given the lengths of the three sides, from which can be calculated the required angles.

(b) Deficient Sides Separated. In Fig. 65 let the bearings of *CD* and *GF* be wanting. Calculate the length and bearing of *FG'* as before. Then, in the triangle *FGG'* there are three sides known from which can be calculated the three angles and therefore the bearings can be deduced.

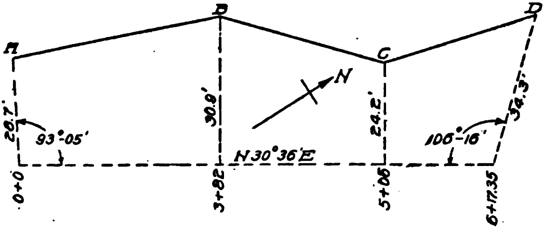


Fig. 67. Illustrating Method of Deducing Bearings and Lengths of Courses

Example. Find the bearings and lengths of the courses *AB*, *BC*, and *CD*, shown in Fig. 67.

Ans. *AB* is N 30° 16′ E, 383.54 ft.; *BC* is N 33° 41′ 34″ E, 124.18 ft.; and *CD* is N 26° 28′ E, 121.27 ft.

Co-Ordinates, or Latitudes and Longitudes of Points. The most accurate and satisfactory method of plotting a traverse in making a map is by co-ordinates. These may easily be computed as follows: Select a north-and-south axis passing through the most westerly point and an east-and-west axis passing through the most southerly point. This is not absolutely necessary but is convenient as it makes all the co-ordinates plus. Using as an example the traverse already given on page 81, the balanced latitude and longitude differences, as discussed under Balancing of Survey, page 83, are given below, and it is seen that *2*, Fig. 68, is both the most southerly and most westerly point in the traverse. The latitude and longitude of this point, therefore, will be taken as zero. From the definition, the latitude of point *3* equals *a3*, which equals the latitude difference of course *2–3;* similarly its longitude equals *3b*, which equals the longitude difference. To get the latitude and longitude of *4*—equal to *c4* and *4d*—simply add to the latitude and longitude of *3* the latitude and longitude differences of course *3–4*, etc., obtaining the figures tabulated above. Expressed as a rule, *the latitude of any point in a traverse is equal to the algebraic sum of the latitude differences from the point selected as origin up to the point in question* and, similarly, *the longitude is equal to the algebraic sum of the longitude differences.*

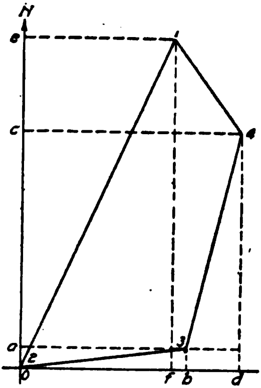


Fig. 68. Illustrating Method of Deducing Latitude and Longitude Differences

Course	Latitude Difference		Longitude Difference		Point	Co-Ordinates	
	N +	S −	E +	W −		Latitude	Longitude
1–2	.....	116.16	.....	44.38	*1*	116.16	44.38
2–3	6.79	.....	58.21	.....	*2*	0.	0.
3–4	80.53	.....	17.19	.....	*3*	6.79	58.21
4–1	28.84	.....	.....	31.02	*4*	87.32	75.40

As a check, the process of successive additions and subtractions continued around to the zero point must always equal zero.

TRANSIT

Forms of Transit. The transit is unquestionably the most important instrument used in the engineering profession and is commonly used by surveyors for measuring horizontal angles. The complete engineer's transit is, in addition, equipped with a compass, a long bubble, a vertical arc, and stadia wires, which make it possible to use a transit in determining bearings, in leveling, and in measuring vertical angles and distances. The construction and adjustment of the simple transit instrument will be explained first, and then the same will be done for each of the attachments.

SIMPLE TRANSIT

Construction of Plain Transit. The plain transit is shown in Fig. 69 and is shown in Fig. 70 separated into its three main parts: upper plate, lower plate, and leveling head. Fig. 69 shows an instrument equipped with a compass which, however, is omitted in Fig. 70.

Upper Plate. The upper plate, Fig. 70(*a*), has fastened to it the two standards which, in turn, support the telescope by means of a horizontal axis. The telescope may thus be revolved about the horizontal axis, or *transited,* as the standards are made high enough to permit this. The telescope itself does not differ in any way from that on the level, "Plane Surveying", Part I, and is simply a telescopic line of sight. Fastened to the upper plate are also two level bubbles placed at right angles to each other and used to level up the plate. The inner vertical axis, or spindle, is rigidly fastened to the center of the upper plate, as shown, and this plate also has two openings cut through it in which are two verniers exactly 180 degrees apart and known as the *A* and *B* verniers. *B* is seldom used.

Lower Plate. The lower plate, Fig. 70(*b*), has on its upper side the graduated circle and is provided with a hollow or outer spindle as shown. The inner spindle fits into the outer spindle and, when in place, the two verniers come into position on the inner edge of the graduated circle. It is thus evident that the upper plate, when in position, is centered and revolves on the lower plate. These two plates may be fastened together and the upper one moved slightly by means of the upper clamp and slow-motion screw. The detail of this construction is shown in Fig. 71(*a*). There is an arm attached to the lower plate which revolves about the axis of the lower plate if the clamp screw is loose, Fig. 70. When this screw is tightened, however, it pushes in

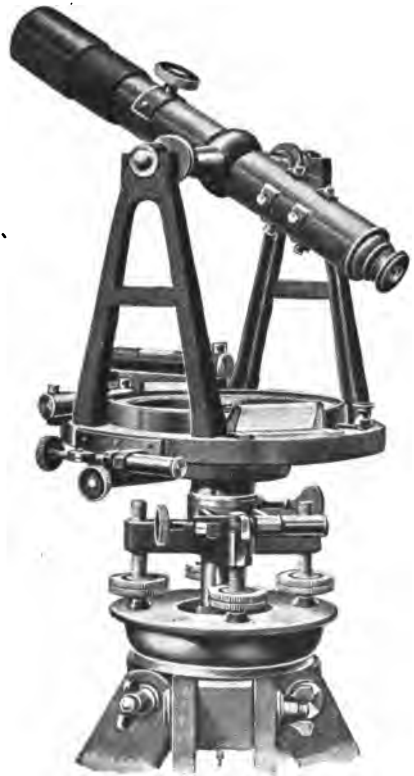


Fig. 69. Engineer's Plain Transit

a small bearing piece at its inner end and clamps the arm firmly to the axis of the lower plate. The end of the clamp arm has a small upward projection which fits in between the upper slow-motion screw and the plunger, as shown in plan in the lower part of Fig. 71(*a*). The plunger is held up tight against the clamp projection, and this, in turn, is held against the end of the slow-motion screw by means of a coiled spring contained in the spring box in which the plunger is placed. When the plates have been

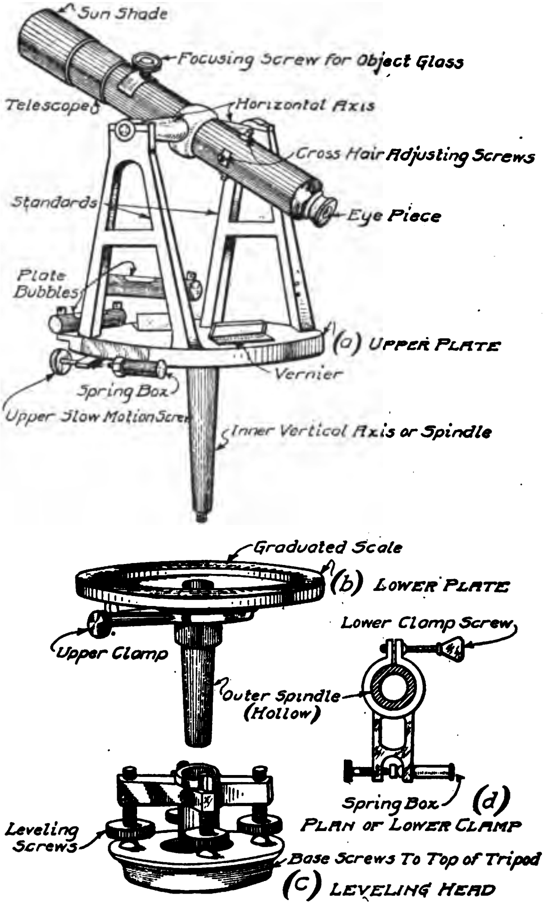


Fig. 70. Main Parts of Plain Transit

clamped with the vernier at as near zero, for example, as can be done by turning them by hand, they may be brought exactly to zero by turning the slow-motion screw; or, when the lower plate is stationary, the upper plate may be turned by hand so the line of sight points nearly on any object, then clamped and brought to point exactly on the object by turning the slow-motion screw, the movement being taken up by the plunger moving in or out of the spring box against the spring.

Leveling Head. The leveling head, Fig. 70(*c*), is essentially the same as that on the wye or dumpy level but, in addition, has what is termed a shifting center. The level is never set up over any particular point, whereas the transit always is. The shifting center is a device to save time in centering the transit, that is, in setting up the instrument so it is exactly over a point on the ground or a stake. For this purpose there is a chain and hook, to which a plumb bob may be fastened, attached to a cap which screws on the lower end of the spindle after the nut which goes on the lower end of the inner spindle and holds the three parts together is in place. In setting up, the instrument is placed as nearly as possible over the mark, the leveling screws are loosened, and the instrument is then shifted on the base until exactly

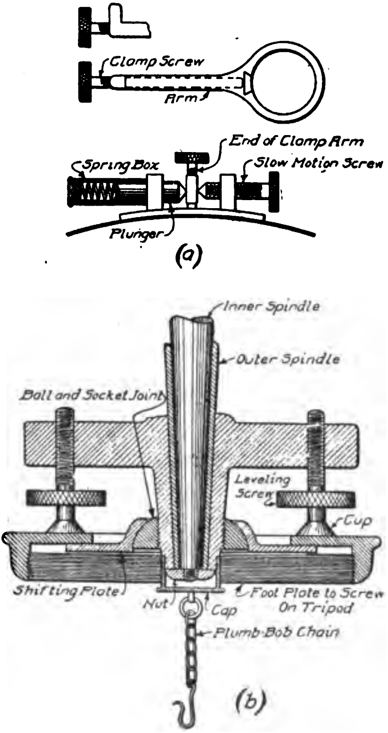


Fig. 71. Details of Plain Transit: (*a*) Upper Clamp and Slow Motion; (*b*) Cross-Section of Leveling Head

centered. The construction of this center is shown in cross-section, Fig. 71(*b*); it can clearly be seen that when the leveling screws have been loosened, the whole instrument may be shifted on the footplate (upper leveling plate), and when they are tightened, the shifting plate is brought up tight against the bottom of the footplate and the instrument is held in position.

There also is provided a clamp and slow-motion screw, by means of which the lower plate may be clamped to the leveling head and moved slightly with reference to it. The details of this construction is shown in Fig. 70, and the device is shown in position in Fig. 69. The clamp screw causes a split collar to clamp the outer spindle, and a projection from the leveling head comes in between the plunger and the slow-motion screw. The action is just the same as for the upper clamp and slow-motion screw. It is possible, for example, to clamp the lower plate in any position and hold it while the upper clamp can be loosened and the upper plate turned in *azimuth*, that is, revolved about its vertical axis, which is the inner spindle. Also, the lower clamp being loose, the upper and lower plates may be set and clamped at any reading and the telescope pointed by hand at any object, the instrument turning about the outer spindle. It may then be clamped and the line of sight brought exactly on the object by means of the lower slow-motion screw.

Directions for Setting Up Transit. *Adjusting Tripod.* Examine the tripod legs and see that they are properly attached to the tripod head and that the screws holding the legs to the head are just tight enough to hold the weight of the leg when it is pulled out to any angle with the head, and no tighter. See that the tripod shoes are tight and, before taking up the instrument, lightly clamp the telescope in a vertical position with the upper and lower plates together, leaving the lower clamp very lightly clamped or loose.

Mounting Transit. Lift the transit out of the box by placing the hand beneath the plates. Avoid lifting it by the telescope or the standards, although when it cannot be handled any other way, the tops of the standards may be grasped firmly between the fingers and thumbs. In attaching it to the tripod, be careful that the threads engage properly and screw it down firmly. Carry the instrument in the most convenient way over the shoulder taking care not to hit it against trees, lamp-posts, doors, etc.

Centering Transit over Stake. To center the transit over a stake, rest one leg of the tripod on the ground, grasp the other legs and place them in the proper directions so the instrument will come approximately over the stake. Now attach the plumb bob to the hook, using a sliding slipknot, Fig. 72(*a*), which may easily be moved up and down so as to lengthen the string and can be untied by simply pulling the free end *A*. Now push the leg *A*, Fig. 72(*b*), firmly into the ground, then pass to leg *B*. Move this leg in or out so that when it is firmly pushed in, the plumb bob comes in line with the stake and the leg *C*. Finally pass around to the leg *C*, and by moving *C* in slightly toward the stake, the bob can be brought very nearly over the stake. Do not try to get it exactly, as the final movement is made by the shifting center.

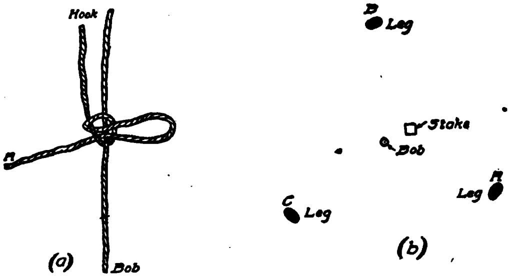


Fig. 72. (*a*) Slipknot for Plumb Bob String; (*b*) Method of Placing Tripod Legs

Leveling Instrument. Level up the instrument approximately. To do this turn the instrument on its vertical axis until the bubble tubes are parallel to a pair of diagonally opposite leveling or plate screws. Now, as you stand facing the instrument, grasp the screws between the thumb and forefinger and turn the thumb of the left hand in the direction the bubble must move. Turn both thumbs in or both thumbs out. Adjusting one tube will disturb the other, but adjust each alternately until the bubble of each is about in the middle. Now loosen the leveling screws and, by means of the shifting center, bring the instrument exactly over the stake and then return to the level screws and do the final leveling up.

The operation of setting up has been described in detail in this way because facility in this particular is very desirable. The whole party is frequently held up while the instrumentman sets up and gets ready for giving the line, etc. In setting up, always watch the leveling head and avoid getting it too much out of level, as it will require considerable movement of the leveling screws to make the plates level if the head is much inclined; and this not only takes time but subjects the screws to unnecessary strain and wear. It is a good plan to shift the leveling base so as to have the long screws over the low head.

TRANSIT OPERATION PROBLEMS

Angular Measurements. *Case 1. Measurement of Horizontal Angle.* Suppose the transit is set up over the point C, Fig. 73. To measure the angle ACB and starting with both upper and lower clamps loose: (a) move the upper and lower plates by hand until the A vernier reads approximately zero; then clamp the upper motion and make the plates read exactly zero by means of the slow-motion screw; (b) turn the instrument by hand until the telescope sights about on A; then clamp the lower motion and set the line of sight exactly on A by the lower slow-motion screw. The instrument is now sighted on A, with the vernier reading zero; (c) loosen the upper clamp; (d) turn the telescope to B, thus moving the upper plate and vernier through the angle ACB as the lower plate is clamped and stationary; clamp the upper motion and set the line of sight on B by the upper slow-motion screw; (e) the vernier reading now gives the angle required. For plate graduations and verniers, refer to the article on Use of Vernier in "Plane Surveying", Part I.

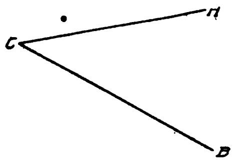


Fig. 73. Measurement of Horizontal Angle or Laying Off Horizontal Angle

Case 2. Reading Angle by Repetition. The reading of the angle by repetition consists in adding together several measurements of an angle and in obtaining the value of the angle by dividing the final reading by the number of repetitions. The method is the same as the preceding except that we continue as follows: (f) leaving the plates clamped as they were when sighted on *B*, Fig. 73, that is, reading the angle *ACB*, loosen the lower motion and sight on *A* again, as in case 1 (a). Follow through (b), (c), and (d), and the vernier will then read twice the angle, etc. It is obvious that for an instrument which reads to 1 minute or to 30 seconds, as most transits do, a single reading of an angle cannot be expected to give an angle closer than plus or minus 20 or 30 seconds. We can read the sum of a number of repetitions just as closely, so, when we divide by the number of repetitions, the error is also divided. One would at first suppose that a very large number of repetitions would give much closer results, but such is not the case, as the principal error then becomes, not the error due to inability to read the plate more closely, but that due to inability to sight the instrument accurately. Remember that 1 second of arc corresponds to a distance of only $\frac{1}{16}$ inch in 1000 feet. Ordinarily, about 6 or 8 repetitions are all that are necessary, and the resulting angle should be within 10 seconds of the correct value.

Case 3. Laying Off Given Angle. The operation of laying off a certain angle is essentially the same as the preceding, except that after the point *A*, Fig. 73, has been sighted and the given angle laid off on the horizontal circle, the tack in the stake *B* must be moved back and forth until it is accurately centered at the intersection of the cross hairs. Also, if it is required to lay off an angle, say, 39° 21′ 45″, very accurately, it is first laid off as just described. The angle thus laid off is only approximately correct, so it is now read by repetition and is found to be, say, 39° 21′ 15″. The distance *CB* equals, say, 500 feet, and as 1 minute of arc equals 0.10 foot in 300 feet, we have 30 seconds equals $(0.10 \div 2)\frac{5}{3}$, or 0.08 foot. If the point *B* is moved away from *A* by 0.08 foot, the angle will be increased by 30 seconds, which makes it 39° 21′ 45″, as required.

Linear Measurement. *Prolongation of Line.* Required to prolong the line *AB*, Fig. 74, and to locate the point *C* with a transit. If possible, this should be done with the instrument at *A*. If *C* is not visible from *A*, then it will be necessary to set up at *B*. In that case, sight at *A*, setting on *A* with either of the slow-motion screws and having both motions clamped. *Transit* the telescope, that is, turn it over about the horizontal axis and set a point in advance. If the line of sight in the telescope is at right angles to the horizontal axis, this point will be the correct point C. If the line of sight is not at 90 degrees to the horizontal axis ($H.A.$), but is at an angle smaller by the amount a, then when the telescope is transited the line of sight will trace in space a conical surface instead of a plane, and the point C, Fig. 74(a), will be obtained to the right of C, the angle C_1BC being $2a$, Fig. 74 being a plan view. Loosen the lower clamp and turn the instrument about 180 degrees in *azimuth*, that is, about its vertical axis, until the telescope again points at A; set on A and clamp. It will be noted that this operation has reversed the position of the horizontal axis end for end and, when the telescope is now transited, a point C_2, Fig. 74(b), will be obtained just as far to the left of C as C_1 was to the right. The distance C_1C_2 is then measured and the correct point C located halfway between C_1 and C_2. This method of prolonging a line, which eliminates errors in the adjustment of the instrument, is known as double hubbing, *double centering*, or double reverse.

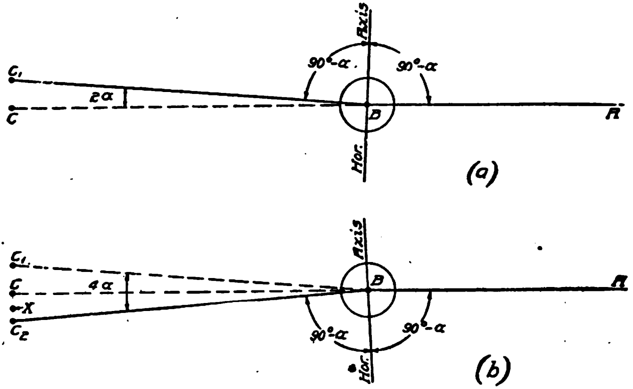


Fig. 74. Method of Double Centering (Used When Transit Is Out of Adjustment)

Transit Adjustments. These fundamental operations with a transit make clear the necessity for the three adjustments of the plain transit, which are: plate bubbles; line of sight; and horizontal axis.

Adjustment of Plate Bubbles. The axes of the plate bubbles should be parallel to the plate, so that the plate will be horizontal when the bubbles are in the middle of their tubes. The constructor of the instrument makes the plates perpendicular to the vertical axis, or spindle. The adjustment of the plate bubbles consists in making the axes of the bubble tubes also perpendicular to the vertical axis of the instrument and hence parallel to the plates, by adjusting the bubble tubes by means of the adjusting screws placed at one end of each tube for that purpose. It depends on the method of reversion, described on page 28, "Plane Surveying", Part I, and is made as follows:

Attach the instrument to the tripod and set up firmly on solid ground, preferably shaded from the sun and protected from the wind. Revolve the transit on its vertical axis so as to bring the bubble tubes parallel to a pair of diagonally opposite leveling screws. Bring the bubble of one of the tubes to the middle by means of these screws. Do the same with the second bubble tube. Adjusting the second tube will throw the first one out, but repeat the alternate operations until each bubble stands in the middle of its tube; in other words, level carefully. Now rotate the instrument on its vertical axis through 180 degrees and note whether each bubble still stands in the middle of its tube. If so the tubes are in adjustment.

If the bubble of either tube runs toward one end, bring it halfway back to the center position by raising the opposite end of the tube by means of the capstan-headed adjusting screw. Relevel the instrument by the leveling screws and again test the tubes. Repeat the operation until the bubbles remain central in the tubes for all positions of the instrument. It is advisable to carry out this adjustment as accurately as possible, as it will facilitate the remaining adjustments. If, after several trials, it is found impossible to adjust the bubbles to the middle of the tubes, either the vertical axis is bent or the plates are sprung, and the instrument should be sent to the maker for correction. If one tube adjusts and the other does not, the fault is in the tube and a new one should be ordered.

Adjustment of Line of Sight. This adjustment is essentially to make the line of sight revolve in a plane or, in other words, to make the line of sight perpendicular to the horizontal axis of the telescope. Having made the first adjustment, as above, carefully level the instrument and clamp the upper motion. Now proceed in the same manner as in connection with prolonging a line, Fig. 74. Drive a stake into the ground about 300 feet ahead of the instrument and drive a tack in the head of the stake. By means of the lower motion revolve the instrument on its vertical axis until the intersection of the cross hairs approximately covers the tack. Now clamp the lower motion and carefully adjust the line of sight on the tack by means of the lower slow-motion screw. Without disturbing either the upper or the lower plates, transit the telescope, that is, revolve it vertically and set a tack in the head of a stake driven into the ground about 300 feet behind the instrument. Carefully adjust the tack to the intersection of the cross hairs. Now unclamp the lower motion and rotate the instrument on its vertical axis until the intersection of the cross hairs again covers the tack in the first stake. Clamp the lower motion, carefully adjust the line of sight by means of the slow-motion screw, again transit the telescope and sight in the direction of the second stake. If the intersection of the cross hairs falls on the tack in the second stake, the line of sight is in adjustment. If it does not, it will have to be adjusted.

In Fig. 74, B is the position of the instrument and A is the forward stake. If the instrument is in adjustment the line of sight, after transiting the telescope and revolving on the vertical axis, should strike the point C. If the instrument is not in adjustment the line of sight, after transiting the telescope, will in the first instance strike some point as C_1. Drive a stake at this point and carefully center a tack. After rotating the instrument on its vertical axis and again transiting the telescope, the line of sight will fall at a point C_2, Fig. 74(*b*), as far on one side of C as C_1 was on the other. Drive a stake at C_2 and carefully center it. Measure the distance C_1C_2 and place a stake at X, one-fourth the distance from C_2 toward C_1. Since the total angle C_1BC_2 is $4a$, the angle XBC will equal a, and if, by moving the cross-hair adjusting screws attached to the cross-hair ring, Fig. 70, the line of sight be moved until it comes on X instead of C_2, its angle will be increased with the horizontal axis by the amount a, making this angle 90 degrees, as required. Repeat the operation of testing the adjustment and correcting the position of the line of sight until the instrument, when sighted on A and transited, strikes C.

It is necessary only that the line of sight shall be accurately in adjustment, but for convenience in using the transit it is also desirable that the vertical cross hair be truly vertical when the instrument is level.

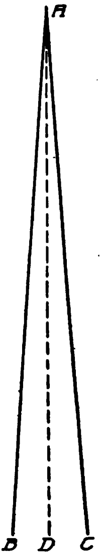


Fig. 75. Diagram to Illustrate Adjustment of Horizontal Axis

The adjustment of the cross hair is made at the same time as that of the line of sight, for any movement of the cross-hair adjusting screws would derange the latter. The method is similar in principle to that for the wye level. When moving the cross hairs so as to bring them to the point X, as described above, also rotate the entire ring slightly, if necessary, so the vertical wire will follow the point X when the telescope is tilted up and down slightly, that is, when the telescope is inclined so the point X comes in the upper and then in the lower part of the field of view. This makes the vertical wire perpendicular to the horizontal axis, and it will therefore be vertical if the horizontal axis is made truly horizontal.

Adjustment of Horizontal Axis. This adjustment is to make the horizontal axis of the telescope perpendicular to the vertical axis of the instrument and is often called the adjustment of the standards, the object given being *to make the standards of equal length.* Really, the object is to make the horizontal axis parallel to the plates or parallel to the axes of the plate bubbles, so that when the latter are in adjustment and have been leveled up, the horizontal axis will be truly horizontal and the line of sight will trace a vertical plane when the telescope is revolved on this axis. This is accomplished by making the horizontal axis perpendicular to the vertical axis and hence parallel to the plate bubble axes, in the following way:

Set up the instrument about 10 feet from a building and level up. Pick out any point that can easily be found again high up on the side of the building, as *A* in Fig. 75. Sight the vertical wire on this point, clamp both motions, turn the telescope down, and mark the point *B* where the line of sight comes. If the left end of the horizontal axis is high, this point will come as shown in Fig. 75. Now loosen the plate and turn the instrument about 180 degrees in azimuth; transit the telescope and sight again on point *A*. The horizontal axis has been reversed in position and, when we plunge the telescope and set a point, it will come at *C* just as far to the right of the true point *D* as *B* was to the left. Bisect *BC* and mark point *D*. Adjust the end of the horizontal axis by trial until the line of sight will cut *D* when the telescope is sighted on *A* and plunged.

The detail of the end of the horizontal axis as usually made is shown in Fig. 76. If the end of the axis is to be lowered, the lock nut on the screw *b* is loosened and then the screw itself. This permits the lower portion *c* of the bearing to move downward, and the upper portion is then brought down by the screws *aa* to a sufficiently tight bearing so that the telescope may easily be turned and will stay in any position. In raising the axis, the operation is exactly the reverse, the screws *aa* being loosened first. Note that the line of sight cannot be brought from *C* to *D* by raising or lowering the axis, and any movement of the axis affects the sighting on *A* more than on the lower point. The adjustment must be made by trial, moving the axis slightly, sighting on *A* and plunging down and repeating until the line of sight comes on *D*.

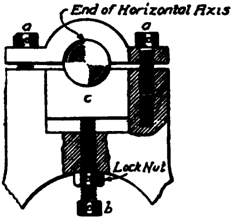


Fig. 76. Detail of Adjustable End of Horizontal Axis

COMPLETE ENGINEER'S TRANSIT

Additions to Plain Transit. One form of an engineer's transit is shown in Fig. 77. It differs from the plain transit in that it is equipped with compass, long bubble, vertical arc, and stadia hairs. The stadia is described later. The compass adjustment is similar to that already described.

Long Bubble. The long bubble tube is attached to the underside of the telescope tube exactly in the same way as the bubble tube on a wye level. By means of this bubble the transit can be used as a level, although this bubble tube is not so sensitive as a wye- or a dumpy-level tube, and the work is of a smaller degree of accuracy. This tube is used also to test the vertical arc and may be used instead of the plate bubbles for leveling the instrument when higher accuracy is necessary. Note also that a clamp and

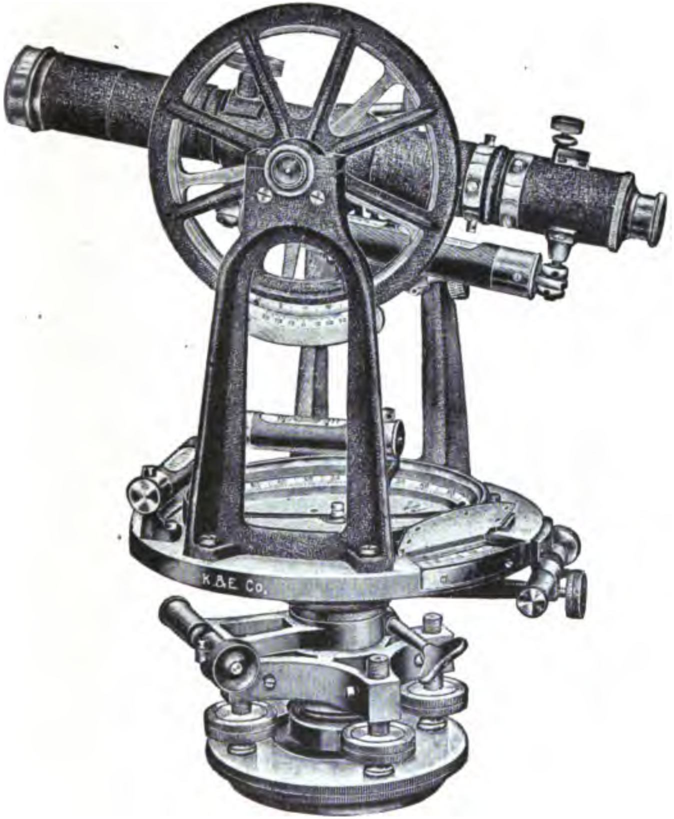


Fig. 77. Engineer's Transit with Vertical Circle and Telescope Level Bubble
Courtesy of Keuffel & Esser Company, New York City

slow-motion screw are attached to the horizontal axis for use with the long bubble and vertical arc. The clamp screw, by means of which the horizontal axis may be clamped and held in any position, is shown at the farther end of the horizontal axis and may be seen through the openings in the vertical arc, Fig. 77. The tangent, or slow-motion, screw with its spring box is shown attached to the farther standard. Sometimes, it is fitted with a graduated head, known as a *gradienter*, which may be used to set off small vertical angles in terms of slope, a certain number of divisions on the gradienter corresponding to a slope of 1 foot in 100 feet, etc.

Adjustment. The necessity for the adjustment of the long bubble tube is obvious, and it is made by the peg method as follows: The only way in which this adjustment differs from the adjustment of the dumpy level, described under Use of Leveling Instruments, "Plane Surveying", Part I, is that in this case the adjustment is made by moving the bubble tube, while in the other case the cross hairs are moved.

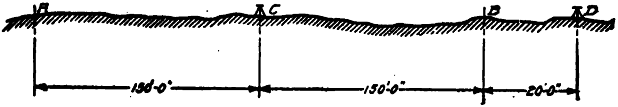


Fig. 78. Peg Method of Adjustment of Long Bubble Tube

To make the axis of the telescope long bubble tube parallel to the line of sight of the telescope, a horizontal line is established; then the line of sight is brought parallel to this line and the bubble is adjusted so as to be in the middle of the tube in the following manner: Select a piece of comparatively level ground, drive a stake, set up the transit over it, and carefully level by the plate levels. Drive two stakes into the ground, one in front of the transit and the other at the same distance behind it. In Fig. 78, *C* shows the position of the transit and *A* and *B* are two stakes, each 150 feet from *C*. There is a fourth stake *D* behind *B* and in line with it from *C*. The transit being leveled by the plate levels, brings the bubble of the telescope tube to the middle of the tube by means of the tangent, or slow-motion, screw attached to the horizontal axis of the telescope. Hold a level rod on *A*, adjust the target to the horizontal cross hair, and note the reading. Unclamp the lower motion, turn the transit on its vertical axis, see that the telescope bubble is on the middle of its tube, and note the reading of a rod held on B. The difference of the readings of the rod held on the two points will give the true difference of level, no matter how much the telescope level may be out of adjustment.

Now take up the transit and remove it to the point D. Carefully level the transit by the plate levels and again bring the bubble of the telescope tube to its middle. Hold the rod on the point B and note its reading. Do the same at the point A and take the difference of the two readings. If the telescope level is in adjustment, this difference will be the same as found when the instrument was over the point C. Otherwise the tube is out of adjustment and may be corrected by the following method:

Method of Correction. Let x represent the difference between the rod readings—that is, the difference of level of A and B—when the transit is at C, Fig. 78; let y represent the difference of level of A and B when the transit is at D; let z represent the difference between x and y.

If y is greater than x, subtract z from the rod reading on A for the transit at D and set the target at this new reading. Revolve the telescope on its horizontal axis, by means of the tangent screw, until the horizontal wire accurately bisects the target. The line of sight is now horizontal. Clamp the telescope axis and bring the bubble to the middle of the tube by means of the capstan-headed screw at one end of the tube. Again hold the rod on B, and then on A, and take the difference of their readings. If this difference now agrees with the true difference of elevation of the two points x, the adjustment is complete. Repeat the operation as often as may be necessary.

If y is less than x, add z to the rod reading on A for the transit at D and set the target at this new reading. Bisect the target by the horizontal cross hair as before; clamp the horizontal axis, and bring the bubble to the middle of the tube. Test and repeat as described before.

The proof of this procedure will not be given here, as it was given in the adjustment of the peg method, but the student is advised to make a sketch illustrating this adjustment.

Vertical Arc. The vernier is similar to that already described, except that it usually is made adjustable, and the arc may be either full 360 degrees or only 180 degrees. It is necessary that the zeros of the vernier and scale coincide when the line of sight is horizontal, and this is accomplished as follows: after adjusting the telescope or long bubble, level up carefully and bring the telescope bubble to the middle of the tube. The line of sight is now horizontal. Loosen the two screws which hold the vernier, shift the vernier so that it reads zero, and tighten the screws fixing the vernier in place.

Distinction between Theodolite and Tachymeter. The complete engineer's transit, sometimes called a tachymeter, should not be confused with the theodolite which is used mainly in the measurement of angles for triangulation in geodetic surveying. The theodolite does not *transit*—revolve in a vertical plane—the telescope being too long and such a motion being unnecessary. Also, it is constructed on a different principle from that of the transit, in as much as no vernier is used on the plates, but the reading of the angle is made by means of micrometer screws. By this device it is possible to read to single seconds of arc and to estimate to 10ths of seconds. This is far closer than it is possible to point the instrument. Hence, instead of the repetition method used with the transit, the direction method is used, that is, a large number of sights are taken on each point, and the average of the scale readings is used. The angle is found by subtracting the scale readings, the scale being fixed in position and not movable like a transit scale.

Transit verniers and scales are made reading to 20 seconds or even to 10 seconds. These are used in city surveying and accurate work, the usual instruments reading only to 1 minute or to 30 seconds.

TRAVERSING WITH TRANSIT

Methods. Three methods may be used in traversing with a transit, namely, by bearings; by angles; and by azimuth.

Traversing by Bearing Method. In traversing by bearings, the compass on the transit is used and the method is the same as for the surveyor's compass, page 73.

Traversing by Deflection Angles. In the common way of traversing by deflection angles the method is as follows: Set up the transit over the point *A*, Fig. 79, and set the verniers at 0. Have the north end of the plate ahead, that is, in the direction of the survey. Without disturbing the upper plate, turn the transit on its lower motion and center on *B*. Lower the needle on its pivot and as soon as it has come to rest take the bearing of the line *AB*. To measure the length of the line, hold one end of the tape or chain directly opposite the transit plumb bob line and send the head man in the direction of *B*. As he reaches the end of his tape, place him accurately in line by the vertical hair. Repeat this operation until the point *B* is reached. Move the transit to the point *B* and set it up as before with the north end of the plate ahead. Transit the telescope and examine the verniers to see if they are at 0. By means of the lower motion center on *A*, lower the needle, and take the bearing. Clamp the lower motion, transit the telescope, and revolve the upper plate so the telescope points to *C*. Read the angle, which is that between *AB* produced and *BC*. Also read the bearing from the compass. Now measure *BC*, as explained before, take up the instrument, and set it up at the point *C*. Continue thus until the series of lines have been surveyed.

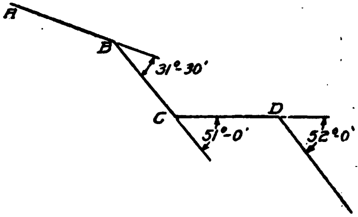


Fig. 79. Illustrating Traverse by Deflection Angles

Compass Check. In work of this character it is desirable to measure the bearings with the compass, as previously described, as a check on the deflection angles. Deflection angles are always marked "right" or "left". Referring to Fig. 79, the records of the survey should be kept as follows, using the left-hand page of the transit or field book and starting from the bottom of the page.

Station	Deflection		Compass
	Right	Left	
7+95	52° 0′	.....	S 52° 45′ E
5+30	......	51° 0′	N 75° 15′ E
2+90	31° 30′	.....	S 53° 45′ E
0	......	.....	S 85° 0′ E

It will be noted that the deflection angles computed from the bearings agree within a reasonable amount with those measured, which show that no big errors have been made, such as reading a degree off or calling an angle right when it should have been left, etc. The deflection angles read with the transit are probably correct to a minute, depending on the least count of the transit vernier, while the compass bearings are only a rough check in comparison, as they may be out 10 minutes or more.

It is evident that if the true bearing of the first course *AB*, Fig. 80, were known, the bearings of all the other courses could be computed, using the measured deflection angles. Supposing the bearing of the first line to be correct, let it be S 85° 0′ E; then as the second line deflects to the right 31 degrees 30 minutes, it is evident that this second line decreases its easting by that amount, so that its bearing will be the difference between 85 degrees and 31 degrees 30 minutes, or S 53° 30′ E. Since the third course deflects to the left by 51 degrees, 0 minutes, its bearing to the east will be increased by this amount but will pass into the northeast quadrant by 14 degrees 30 minutes, making its bearing 90 degrees minus 14 degrees 30 minutes, or N 75° 30′ E. The fourth course deflects to the right 52 degrees 0 minutes, returning to the southeast quadrant by 37 degrees 30 minutes, making its bearing 90 degrees minus 37 degrees 30 minutes, or S 52° 30′ E.

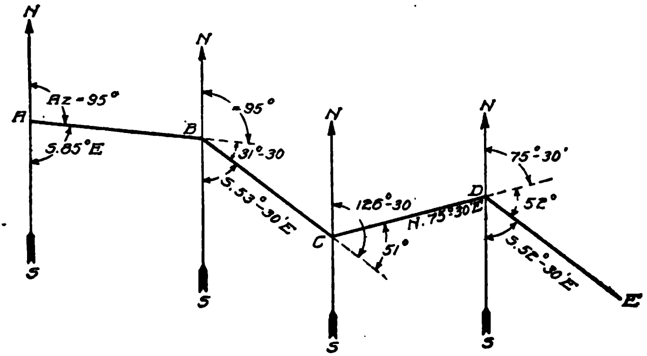


Fig. 80. Illustrating Deduction of Bearings from Deflection Angles

Traversing by Interior or Exterior Angles. A traverse may also be run by interior or exterior angles. Thus, for the traverse shown in Fig. 79, we may read the angle *CBA*, angle *DCB*, etc. Here it is advisable to measure the bearings, as a check, in case there should be any doubt as to whether the right-hand or left-hand angle at each station has been measured. Accurate work in plane surveying is usually done in this way. Thus both the exterior and interior angles at each station or in an open traverse, the left-hand and right-hand angles, are measured by repetition. If they add up to within 10 or 15 seconds of 360 degrees, the field work of measuring the angles would ordinarily be considered satisfactory and the error would be distributed equally between the two angles so as to make them total 360 degrees, and this value would be used in the computations. The bearing of each line may be computed, provided the bearing of one line is known.

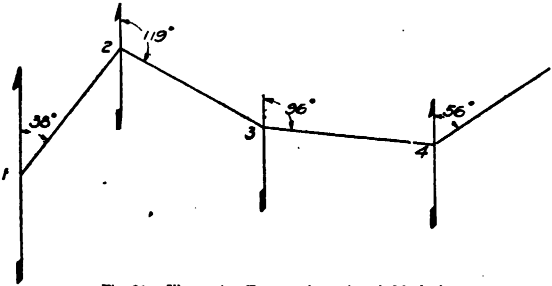


Fig. 81. Illustrating Traverse by Azimuth Method

Traversing by Azimuth Method. *Azimuth Angle.* The azimuth of a line is the angle measured from a zero direction, which is usually north, clockwise, around to the line in question. The value of the azimuth angle, therefore, may be anything between 0 and 360 degrees.

Deduction of Bearings from Azimuths. Azimuths may easily be changed to bearings. Suppose, for example, that in Fig. 81 the arrow represents the zero azimuth direction and is true north. Then, since the first line is in the northeast quadrant, its bearing N 38° E will be the same as its azimuth, 38 degrees. The second line, however, passes from the northeast into the southeast quadrant and its azimuth is, as noted in Fig. 81, 119 degrees. Its bearing, however, will be 180° − 119°, or S 61° E. The third line swings back 23° but still remains in the southeast quadrant and its azimuth therefore will be 96 degrees. Its bearing will be S 84° E. In a similar manner, the bearing of the fourth line may be deduced from the azimuth. The relation between deflection angles, azimuths, and bearings is thus a very simple one.

Fig. 82 illustrates a traverse beginning in the southeast quadrant. The bearing of the first line is S 41° E, therefore its azimuth will be 180 degrees minus 41 degrees, or 139 degrees. The bearing of the second line is S 73° E, and its azimuth therefore will be 107 degrees; and so on with the remaining courses of the traverse.

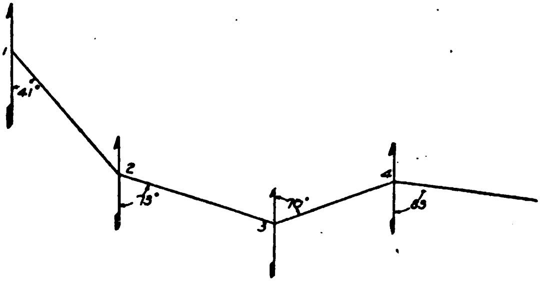


Fig. 82. Illustrating Method of Determining Azimuths

By a similar process of reasoning the azimuths of courses in the southwest and northwest quadrants may be deduced from the bearings, and *vice versa.*

Determination of Azimuths. The limiting values of azimuths are as tabulated below.

Quadrant	Bearing	Azimuth
Northeast	......	Bearing
Southeast	......	Bearing Supplement
Southwest	......	180° + bearing
Northwest	......	360° − bearing
....	North	0 or 360°
....	East	90°
....	South	180°
....	West	270°

Azimuths are usually measured with a transit. For convenience in determining azimuths, the inside graduations of the horizontal circle are numbered from 0 to 360 degrees to the right, beginning at the north end. The outside graduations are numbered from 0 to 360 degrees to the left, so that an angle may be read either toward the right or the left.

In traversing by the azimuth method it is necessary that the plates of the transit be set and clamped so that the vernier will read zero when the line of sight is pointing in the direction of the zero azimuth—this is almost always taken as true north. In land surveying, where accuracy is important, a true north direction must be determined. As a transit is used for the work, it is not sufficiently accurate to use a compass to determine this true north, but more accurate methods must be used, such as the solar method or by observation on the North Star as described in "Plane Surveying", Part III. A few surveyors use the south direction for zero azimuth but the north is preferable.

Having determined the position of the lower plate, if the telescope is directed at any point, the vernier will give its azimuth. As the direction of north is determined at only one point in the survey, it is necessary when setting up at any other station to revolve the lower plate and clamp it in such position that the zero line is parallel to the original zero azimuth line. This is done as follows:

First lay out a meridian through station *1*, Fig. 81, and define it by a stake driven 300 or 400 feet toward the north. To begin the survey, carefully center the transit over station *1*, with verniers set to zero; turn the instrument on its lower motion until the line of sight approximately covers the stake at the north end of the meridian and carefully center it by the lower tangent screw; lower the compass needle, if there is one, and note and record the magnetic declination. With the lower motion securely clamped, unclamp the upper motion and revolve the upper plate in the direction of station *2*. Clamp the upper motion and carefully center the line of sight by the upper tangent screw. Note and record the angle as read on the plate, which will be the azimuth of the line *1–2;* note and record the compass bearing, as a check, and measure the distance from *1* to *2*.

Now, with the upper motion securely clamped and the lower one loose, remove the transit to station *2* and carefully center it over the stake, with the north end of the plate ahead, that is, in the direction from *1* to *2*. Check the reading on the plate to see that it has not changed while carrying the instrument from *1* to *2*. Transit the telescope, unclamp the lower motion, and bring the line of sight to cover station *1*. Carefully center it by the lower tangent screw. Clamp the lower motion and transit the telescope. The line of sight is now pointing in the direction of the line *1–2* prolonged and, as the plate reads the azimuth of the line *1–2*, it evidently must be in a position parallel to that which it had at station *1* and, if the telescope were turned until the plate read zero, it would be pointing north. All that is necessary then is to unclamp the upper motion, revolve the upper plate until the line of sight falls on station *3*, and carefully center it by the upper tangent screw. Read and record the plate angle, which will be the azimuth of the line *2–3*. Read the bearing of the needle for a check and measure the distance from *2* to *3*.

With the upper plate securely clamped, and lower motion free, move the instrument to station *3* and proceed as before; and so continue throughout the traverse.

Checking Traverses. *Closed Traverse.* When a closed traverse is run, the angles may be checked by the geometrical rule that the sum of the interior angles must be equal to 180°(n–2), n being the number of sides. For a 4-sided figure the sum of the interior angles should equal 360 degrees; if they do not they would be adjusted by distributing the error equally among them. If the traverse has been run by azimuth in the manner described above, it evidently is important to have the line of sight exactly at right angles to the horizontal axis, and this adjustment should be made carefully. In a closed traverse, as shown in Fig. 58, supposing a meridian were determined at station *1*, the azimuth of the course *1–2* and also of *1–6* would be read when the instrument was at station *1*. Proceeding as described above, the instrument would be moved to stations *2*, *3*, *4*, *5*, and finally to *6*, and at each one of these stations the plate would be oriented, or set parallel with its position at station *1* by taking a back sight on the previous station. When the instrument is at *6*, therefore, we back sight on *5* and may then read the azimuth of *6–1*. This should check, allowing for the 180-degree difference, with the azimuth of *1–6*. It seldom will check exactly owing to slight errors made in setting the plate at the previous stations. The amount by which it fails to check is a measure of the accuracy of the work. In general, for azimuth work of this kind it should not exceed the least reading of the instrument $\frac{s}{n}$ times the square root of the number of stations ($\frac{s}{n}$ is defined on page 26, "Plane Surveying", Part I). If the error is within a satisfactory limit, it is divided by the number of stations, and the azimuth at the first station is adjusted by this amount and at the second by twice this amount, etc., so that the azimuths would close exactly.

Open Traverse. As has been noted, there is no absolute check on an open traverse. As a check against gross errors, it is always advisable to read the magnetic bearings of the lines, or courses, and to compare these with the bearings computed from the angles or azimuths, as the case may be. It is also possible to take another observation, by the solar or star method, after a traverse has been run for some distance, and in this way obtain a check on the direction as computed or carried on from the starting point. Open traverses are used only in such work as railroad, road, and canal surveys. In land surveying the traverse is closed back on the starting point.

Convergence of Meridians. It is well to note that meridian lines are parallel only at the equator. This is due to the fact that the earth is round and meridians converge from the equator toward the earth's poles, where they all meet. If a traverse is run from an observed meridian for, say, 1 mile in an east-and-west direction in latitude 50°, and then another observation is taken, the meridian thus determined will not be parallel to the first but will incline toward it, and in order to use it as a check a correction shown in Table II, must be applied.

Example. An open traverse is run from an observed azimuth at a point *A* for distance of 3 miles. The final point *K* is 2 miles east of the starting point and the average latitude of the traverse is 40°. An observation for meridian is taken at *K*, which gives the azimuth of the east course *J–K* as

TABLE II

Convergence of Meridian Lines

Convergence		
Latitude (degrees)	Per Mile (minutes)	Per Minute (feet)
10	0.153	34525
20	0.316	16731
30	0.500	10552
40	0.727	7265
50	1.032	5118
60	1.499	3523

32° 22′, whereas the value from the survey is 32° 20′. What is the actual error in the survey azimuth?

Ans. The meridian at *K* would converge toward that at *A* by 0.727×2, or about 1′ 30″. This would make the azimuth of the course *J–K* from the observation too big by this amount, hence the error in carrying forward the azimuth from *A* is only 30″.

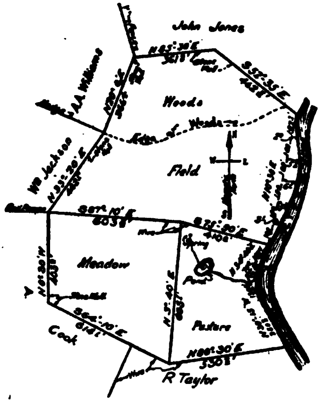


Fig. 83. Sketch Illustrating Field Notes

Selection of Method of Traversing. In surveys with a compass, bearings are used; in farm or suburban land surveys with a transit, azimuths are used; in city land surveys, interior and exterior angles, measured by repetition and adjusted, are used; and in railroad, road, and canal surveys, either deflection angles or azimuths are used.

Method of Keeping Notes. All notes of measurements of angles and distances, as soon as made, should be recorded in a special notebook adapted to the purpose. Avoid the practice of making notes on scraps of paper and in small pocket notebooks and of filling in details from memory. The notes probably will be used by other persons who are miles distant and entirely unfamiliar with the locality for plotting and for general information. These persons must depend upon the record and the manner in which it is recorded for their interpretation. To this end the notes should be clear, accurate, and concise, with enough detail to give all necessary information. They should permit only one interpretation, and that the correct one.

The note keeper should constantly bear in mind the nature of the survey and the object to be attained. This will enable him to determine what measurements are necessary. Do not crowd the notes. Use the left-hand page of the book for notation and the right-hand page for date, names of party and positions, and such sketches and remarks as may be necessary. The record usually is made with a pencil, using a medium hard grade. If incorrect entries are made, neatly erase them. Primarily avoid errors as erasures tend to discredit the work. After each entry in the notebook, index it with the subject of the survey, the dates between which the record was made, the file number of the volume, and all other information that may be of service when filed for future reference. Above all, do not lose a notebook, as it may contain information that cannot be recovered at any price.

Fig. 83 shows a sketch which is more complete in the matter of distances and angles than is usual, most of such records being put in the notes. The usual forms for notes have already been given.

USE OF STADIA

Principle of Stadia Operation. Attached to the ring carrying the horizontal and vertical hairs are two auxiliary horizontal hairs called *stadia hairs* or *wires*. These hairs may be either in a fixed position or adjustable. Adjustable hairs are seldom used, the fixed hairs, Fig. 84, being better and much less expensive. Any instrument maker will equip a level or a transit with stadia wires, and they should be included in every outfit.

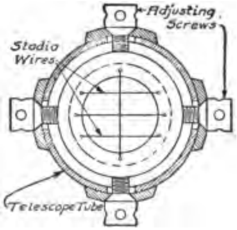


Fig. 84. Sketch Illustrating Arrangement of Stadia Hairs

The stadia is used for measuring horizontal distances and differences of elevation without the use of chain or tape or other apparatus except the leveling rod or a stadia rod. Thus, on looking through the telescope at a rod held 100 feet away, it will be noted that the distance on the rod intercepted between the two stadia wires is practically 1 foot. With the rod 200 feet away, it will be about 2 feet, and for any other distance it will be in direct proportion.

Unfortunately, the construction of the telescope of an engineering instrument modifies the above simple statement, and a formula for the use of the stadia will now be deduced.

Referring to Fig. 85, let the intercept on the rod be S, the distance between the stadia wires i, the distance from the object glass to the center of the instrument c, and the other dimensions as shown, the telescope being assumed horizontal. It is evident that the lines drawn from the stadia wires through the center of the object glass, when prolonged, will cut the rod as shown at A and B, and, on looking through the telescope, the wires will appear to be at A and B, and the intercept on the rod will be AB or S. The required distance is

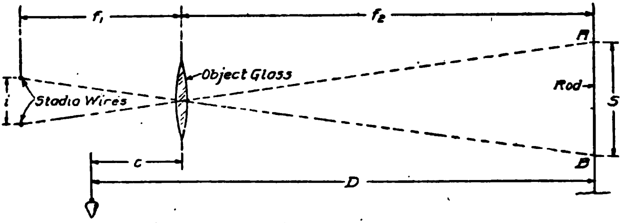


Fig. 85. Diagram Illustrating Stadia Principle

$$D = f_2 + c \qquad (1)$$

Of course, c will vary with each sight, as the objective is moved in and out to focus on the rod. For all practical purposes it can be taken as constant, however, as the stadia is not used to get distances with great accuracy, seldom closer than one foot, and c will vary by only a few 100ths.

To get a practical formula, therefore, an expression must be derived for f_2. From similar triangles, we have

$$i : S : : f_1 : f_2$$

hence

$$f_2 = \frac{f_1 S}{i} \qquad (2)$$

Here S and i can be measured but it would be difficult to measure f_1 with sufficient accuracy in the field; therefore we apply the well-known principle of optics which may be stated as

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f} \tag{3}$$

in which f is the focal length of the lens—the objective, in this case.

A student may easily verify this law by experimenting with an ordinary reading or magnifying glass. Find f by allowing the sun's rays to pass through the lens and to come to a focus on a piece of paper. If the lens is strong enough, it will burn the paper when properly focused and the distance from the center of the lens to the paper will equal f. Now place the lens 3 or 4 feet from a window and hold a piece of paper behind it, moving the paper in or out until a clear but inverted image of the window is formed. Measure the distances from the window to the lens f_2 and from the lens to the paper f_1; these measurements will be found to satisfy equation (3). Therefore, from equation (3), we have

$$f_1=\frac{ff_2}{(f_2-f)}$$

substituting this value in equation (2), there results

$$f_2=\frac{f}{i}S+f$$

which, substituted in equation (1), gives

$$D=\frac{f}{i}S+(f+c) \tag{4}$$

Constants. The ratio $\frac{f}{i}$ is called the *stadia constant* and, in making an instrument, the maker tries to make this ratio exactly 100. Its exact value may be found as described below but, for all ordinary purposes, it can be taken as 100. The quantity $f+c$ is sometimes called the *addition constant,* although, as we have already seen, c is not actually a constant. Its average value is about 1 foot, and the value for each instrument is usually furnished by the maker.

Determination of Stadia Constant. To determine the stadia constant, first find the value of the addition constant, $f+c$, as follows: Sight the telescope on a very distant point and measure the distance between the objective and the cross hairs, which will practically equal f. Now sight on a point about 400 feet away and measure the distance from the center of the instrument to the object glass. This is the average value of c.

To determine $\frac{f}{i}$, select a piece of ground that is nearly level for 400 or 500 feet. Set up the instrument at one end, measure a distance equal to $f+c$ forward from the plumb bob, and put in a stake. Lay off successive lengths of 100, 200, 300 feet, etc., in front of this stake, marking each 100-foot length. Level the instrument and, holding a rod on each point, measure the intercepts on the rod as S_1, S_2, etc. Now the distance from the center of the instrument to the 100-foot stake is

$$D_1 = 100 + (f+c), \text{ etc.}$$

But D_1 may also be found from equation (4), in which

$$D_1 = \frac{f}{i} S_1 + (f+c)$$

Hence, by equating these two values, we have for the first stake

$$\frac{f}{i} = \frac{100}{S}$$

and for the second stake

$$\frac{f}{i} = \frac{200}{S}, \text{ etc.}$$

giving several values of the constant, which may be averaged up and the mean used. As already noted, this usually will be very close to 100.

Inclined Readings. In actual practice it is very seldom that stadia readings are taken with the telescope horizontal; indeed, the stadia is used principally in cases where both the horizontal distance (*H.D.*) and the vertical distance, or difference in elevation (*D.E.*), are required. When the line of sight is inclined, equation (4) must be modified.

In taking readings with the telescope inclined, Fig. 86, the distance AE from the horizontal axis to the top of the stake over which the instrument is set up is first measured. It is called the height of instrument (*H.I.*). Suppose *B* to be any point, the horizontal distance and the difference in elevation from *A* being required. If the telescope is inclined so that the center wire cuts a rod held vertically on *B* at the point *c*, such that the height *cB* equals the height of the instrument, then the vertical arc on the instrument will read the vertical angle *a*. Also read the intercept between *b* and *e*, or *I*. If we knew the distance *ad*, it would correspond to *S* in equation (4), since it is at right angles to the line of sight, and this equation would then give the inclined

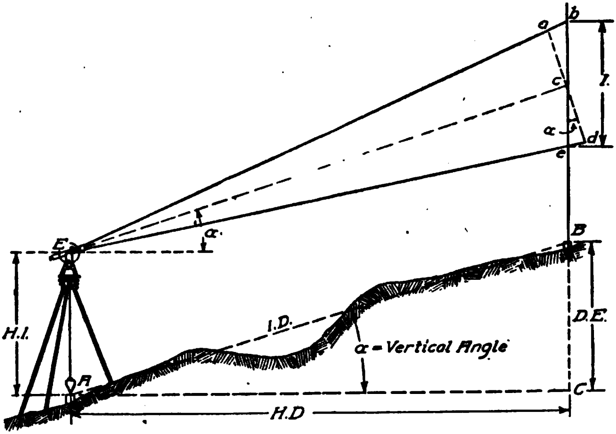


Fig. 86. Diagram Illustrating Method of Inclined Stadia Readings

distance (*I.D.*) equal to *Ec*, from which the height of the instrument and the difference in elevation could be found, knowing *a*. Assuming the angle at *a* in the triangle *abc* and that at *d* in *cde* to be right angles, it is seen from the diagram that

$$ad = I \cos a$$

Hence, from equation (4), we have

$$I.D. = \frac{f}{i} I \cos a + (f + c) \tag{5}$$

But $\quad H.D. = I.D. \cos a$

and $\quad D.E. = I.D. \sin a$

Therefore, we have

$$H.D.=\frac{f}{i}I\cos^2 a+(f+c)\cos a \qquad (6)$$

and

$$D.E.=\frac{f}{i}I\sin a\cos a+(f+c)\sin a \qquad (7)$$

The use of these formulas is greatly simplified by means of tables, diagrams, or special stadia slide rules. A diagram for this purpose is found under Topography by Transit and Stadia, in "Plane Surveying", Part IV. Table III, given at the end of this section, also is used for stadia deductions, as in the following example.

Field Notes for Stadia Surveys. Herewith is shown an example of the method of keeping notes on a stadia survey.

Instrument at Station *4*, Elevation, 633.5′, *H.I.*, 5.1′

Sight To	Azimuth		Vertical Angle		*I.*	*H.D.*	*D.E.*	Elev.	Remarks
	(°)	(′)	(°)	(′)					
△*5*	152	51	−0	11	5 232	5 241	−1.7	631.8	Bear. S 27° 0′ E
⊙*1*	89	25	+5	13	5.24	5 210	+47.4	680.9	
2	85	18	+5	48	3.55	5.253	+35.8	669.3	
3	90	6	+5	27	2.93	2.910	+27.8	661.3	
4	106	35	+1	23	2.35	2.360	+5.7	639.3	
5	114	50	+0	33	2.45	2.460	+2.3	635.8	
6	132	33	−7	52	2.23	2.197	−30.3	603.2	
7	152	57	−18	55	2.28	2.050	−70.2	563.3	
8	75	4	+5	20	2.77	2.750	+25.7	659.2	

In this particular case a traverse has been run with the transit by azimuth, using the stadia to determine the lengths. The instrument is at station *4* of the survey. The height of the instrument has been measured with a rod or tape and, following the method described for azimuth traversing, a back sight has been taken on station *3* and the azimuth of the course *4–5* determined as 152 degrees 51 minutes. Using the height of instrument of 5.1 feet, the vertical angle is obtained as described above for inclined readings, and the stadia interval *I.*, which is 5.232 feet, also is carefully read from the rod held on station *5*. As a check on the azimuth, the bearing of the course in this case also was read and recorded in the remark column. The data in columns headed *H.D.*, *D.E.*, and Elev. are computed from the field measurements, namely, the vertical angle and the stadia interval by equations

(6) and (7). The sights marked by circles and numbered *1*, *2*, *3*, etc., following the sight to station *5*, denoted by a triangle as being a main station, are side shots from station *4*. In other words, when the instrument was at *4*, it was necessary to locate eight points in the vicinity of *4*, required in the survey. These points are located by reading the azimuth, vertical angle, and stadia just the same as for station *5* but usually without so great care, as they are not main points in the traverse.

Reduction of Stadia Readings, Using Table III. *Difference in Elevation.* For the difference in elevation, equation (7) is used. Instead of looking up the natural sine and cosine of the vertical angle and substituting them in the formula, use the values given in Table III. For example, for the work shown in the notes above, $\frac{f}{i}$ was measured and found to be 100 and $(f+c)$ was 0.9 foot. For the stadia reading for side shot *2*, the vertical angle is 5 degrees 48 minutes, for which the natural sine equals 0.1011, and the natural cosine equals 0.9949. Hence, from equation (7)

$$D.E. = (100 \times 3.55 \times 0.1011 \times 0.9949) \times (0.9 \times 0.1011)$$

From Table III, however, we find in the column headed Diff. Elev., for 5° 48′, the value 10.05, which is $100 \times 0.1011 \times 0.9949$, or $100 \sin a \cos a$. Also, at the foot of the Diff. Elev. column is given the value of $(f+c) \sin a$ for three values of $f+c$. Interpolating for $f+c=0.9$, we get 0.09. Hence, for side shot *2*, the $D.E. = (3.55 \times 10.05) + 0.09$, or 35.8 feet, thus reducing the computation. In a similar manner, using Table III, we find for side shot *6* that the $D.E. = (2.23 \times 13.56) + 0.12$, or 30.3 feet, etc.

Horizontal Distance. The computation of the horizontal distance is very similar, except that, to save multiplying by a large number, the table gives the correction to be subtracted from $100 \times I$. to obtain the correct distance. For example, using side shot *2* and equation (6), we have

$$H.D. = [100 \times 3.55 \times (0.9949)^2] + (0.9 \times 0.9949)$$

Now $100 \times (0.9949)^2 = 98.98$, and it is easier to do the multiplication shown above in this way—$H.D. = (3.55 \times 100) - [3.55\ (100 - 98.98)] + (0.9 \times 0.9949)$—than the multiplication shown above. In other words, for side shot *2*, the $H.D. = 3.55 - (3.55 \times 1.02) + 0.9$, or 352.3 feet. Time can again be saved by taking these values directly from Table III. Thus, in the column headed Hor. Corr., for 5° 48′ will be found 1.02 as used above, and at the foot of the column is given the value of $(f+c)\cos a$ for three values of $f+c$, which shows that, in this case, by interpolation for $f+c=0.9$, the value will be 0.9.

In a similar manner we have for side shot *6*, using Table III

$$H.D. = 223-(2.23\times 1.87)+0.89 = 219.7 \text{ feet, etc.}$$

Closeness of Computations. It is usual to carry the computations of all differences in elevation to 10ths, and when added or subtracted—as the sign of the vertical angle may indicate—to the elevation of the transit station, the elevation of the point sighted to is obtained. Traverse sides also are computed to 10ths, but side shots seldom are computed closer than the nearest foot.

By means of Table III, students should compute, for practice, the values for horizontal distance and difference in elevation for the other points given in the above notes but not computed. The results should check the values given.

Stadia Rods. For short sights a self-reading, or Philadelphia, level rod may be used, but for longer sights the marks on these rods become very indistinct and a special rod is used. A large number of designs have been made and used, but the most satisfactory as well as the simplest is that shown in Fig. 87. This marking is shown in four sections and represents the graduations for a 12-foot rod. The marks and numbers at each foot are in red. The distance between two points is 0.10 foot; so the rod can be read to 0.05 foot directly, and hundredths may easily be estimated. The numerals on this rod are especially designed to avoid mistaking the 5 for a 3, for example, and it also has the advantage of the sharp contrast between the black and the white, which makes it possible to see the markings clearly on a cloudy day and at a distance of 1500 feet or more.

Stadia rods usually are made in the same way as the one-piece level rod but are sometimes jointed or hinged for convenience in transportation. Some offices make a tracing of the rod markings, as shown in Fig. 87, to actual scale, and have black prints made on heavy paper. The numerals are then painted in in red, and the strips are cut out and nailed to a board, properly registered end to end. In this way a cheap rod, costing only a few cents, may be quickly made and may be discarded when the survey is finished.

Many surveyors specially graduate their rods so that the stadia wires will intercept one large division when the rod is held at a distance of $100+(f+c)$ feet from the instrument. This means that the large division will be greater or less than 1 foot, depending on whether the stadia constant is greater or less than 100. Instrument makers now place the stadia hairs so accurately, however, that no appreciable error will occur if $\frac{f}{i}$ is taken as 100 and a rod graduated in feet is used. On the other hand, if the constant does differ much from 100, this method of graduating the rod saves in computation, as the rod graduations are made to suit the constant, which therefore may be taken as 100.

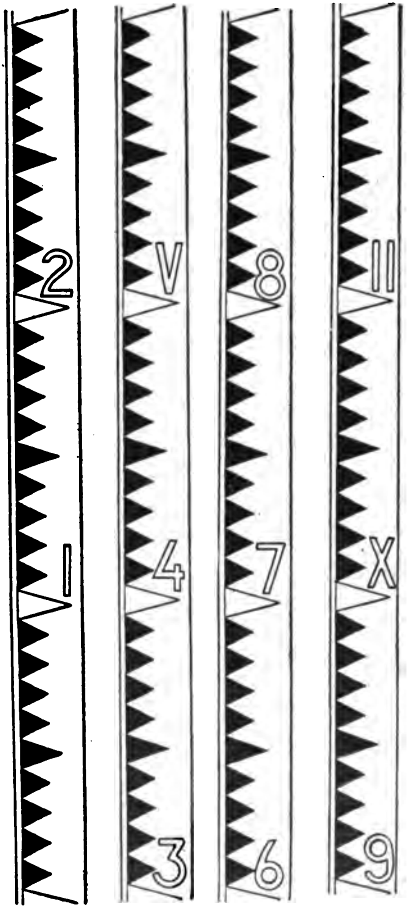


Fig. 87. Stadia Rod

Adjustable Stadia Hairs. Adjustable stadia hairs are seldom used. The distance between the stadia hairs may be adjusted by two additional adjusting screws placed at the top and bottom of the tube just back of the cross-hair adjusting screws. A rod graduated to feet is placed at a distance of, say, $100+(f+c)$ feet from the instrument, and the hairs are adjusted by means of these screws so as to intercept exactly 1 foot.

Use of Stadia. The stadia furnishes a very rapid method for topographic surveying, for which it is used extensively. It can also be used to advantage for rough farm surveys, particularly in rough but unwooded country.

TABLE III

Horizontal Distances and Elevations from Stadia Readings

Minutes	0°		1°		2°		3°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	0.00	0.00	0.03	1.74	0.12	3.49	0.27	5.23
2	0.00	0.06	0.03	1.80	0.13	3.55	0.28	5.28
4	0.00	0.12	0.03	1.86	0.13	3.60	0.29	5.34
6	0.00	0.17	0.04	1.92	0.13	3.66	0.29	5.40
8	0.00	0.23	0.04	1.98	0.14	3.72	0.30	5.46
10	0.00	0.29	0.04	2.04	0.14	3.78	0.31	5.52
12	0.00	0.35	0.04	2.09	0.15	3.84	0.31	5.57
14	0.00	0.41	0.05	2.15	0.15	3.90	0.32	5.63
16	0.00	0.47	0.05	2.21	0.16	3.95	0.32	5.69
18	0.00	0.52	0.05	2.27	0.16	4.01	0.33	5.75
20	0.00	0.58	0.05	2.33	0.17	4.07	0.34	5.80
22	0.00	0.64	0.06	2.38	0.17	4.13	0.34	5.86
24	0.00	0.70	0.06	2.44	0.18	4.18	0.35	5.92
26	0.01	0.76	0.06	2.50	0.18	4.24	0.36	5.98
28	0.01	0.81	0.07	2.56	0.19	4.30	0.37	6.04
30	0.01	0.87	0.07	2.62	0.19	4.36	0.37	6.09
32	0.01	0.93	0.07	2.67	0.20	4.42	0.38	6.15
34	0.01	0.99	0.07	2.73	0.20	4.48	0.38	6.21
36	0.01	1.05	0.08	2.79	0.21	4.53	0.39	6.27
38	0.01	1.11	0.08	2.85	0.21	4.59	0.40	6.33
40	0.01	1.16	0.08	2.91	0.22	4.65	0.41	6.38
42	0.01	1.22	0.09	2.97	0.22	4.71	0.41	6.44
44	0.02	1.28	0.09	3.02	0.23	4.76	0.42	6.50
46	0.02	1.34	0.10	3.08	0.23	4.82	0.43	6.56
48	0.02	1.40	0.10	3.14	0.24	4.88	0.44	6.61
50	0.02	1.45	0.10	3.20	0.24	4.94	0.44	6.67
52	0.02	1.51	0.11	3.26	0.25	4.99	0.45	6.73
54	0.02	1.57	0.11	3.31	0.26	5.05	0.46	6.78
56	0.03	1.63	0.11	3.37	0.26	5.11	0.47	6.84
58	0.03	1.69	0.12	3.43	0.27	5.17	0.48	6.90
60	0.03	1.74	0.12	3.49	0.27	5.23	0.49	6.96
$(f+c)=0.75$	0.75	0.01	0.75	0.02	0.75	0.03	0.75	0.05
$(f+c)=1.00$	1.00	0.01	1.00	0.03	1.00	0.04	1.00	0.06
$(f+c)=1.25$	1.25	0.02	1.25	0.03	1.25	0.05	1.25	0.08

TABLE III

(Continued)

Horizontal Distances and Elevations from Stadia Readings

Minutes	4°		5°		6°		7°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	0.49	6.96	0.76	8.68	1.09	10.40	1.49	12.10
2	0.49	7.02	0.77	8.74	1.10	10.45	1.50	12.15
4	0.50	7.07	0.78	8.80	1.12	10.51	1.52	12.21
6	0.51	7.13	0.79	8.85	1.13	10.57	1.53	12.26
8	0.52	7.19	0.80	8.91	1.14	10.62	1.54	12.32
10	0.53	7.25	0.81	8.97	1.15	10.68	1.56	12.38
12	0.54	7.30	0.82	9.03	1.16	10.74	1.57	12.43
14	0.54	7.36	0.83	9.08	1.18	10.79	1.59	12.49
16	0.55	7.42	0.84	9.14	1.19	10.85	1.60	12.55
18	0.56	7.48	0.85	9.20	1.20	10.91	1.61	12.60
20	0.57	7.53	0.86	9.25	1.22	10.96	1.63	12.66
22	0.58	7.59	0.87	9.31	1.23	11.02	1.64	12.72
24	0.59	7.65	0.89	9.37	1.24	11.08	1.66	12.77
26	0.60	7.71	0.90	9.43	1.26	11.13	1.67	12.83
28	0.61	7.76	0.91	9.48	1.27	11.19	1.69	12.88
30	0.62	7.82	0.92	9.54	1.28	11.25	1.71	12.94
32	0.62	7.88	0.93	9.60	1.29	11.30	1.72	13.00
34	0.63	7.94	0.94	9.65	1.31	11.36	1.73	13.05
36	0.64	7.99	0.95	9.71	1.32	11.42	1.75	13.11
38	0.65	8.05	0.96	9.77	1.33	11.47	1.76	13.17
40	0.66	8.11	0.97	9.83	1.35	11.53	1.78	13.22
42	0.67	8.17	0.99	9.88	1.36	11.59	1.80	13.28
44	0.68	8.22	1.00	9.94	1.37	11.64	1.81	13.33
46	0.69	8.28	1.01	10.00	1.39	11.70	1.83	13.39
48	0.70	8.34	1.02	10.05	1.40	11.76	1.84	13.45
50	0.71	8.40	1.03	10.11	1.42	11.81	1.86	13.50
52	0.72	8.45	1.04	10.17	1.43	11.87	1.87	13.56
54	0.73	8.51	1.06	10.22	1.44	11.93	1.89	13.61
56	0.74	8.57	1.07	10.28	1.46	11.98	1.90	13.67
58	0.75	8.63	1.08	10.34	1.47	12.04	1.92	13.73
60	0.76	8.68	1.09	10.40	1.49	12.10	1.94	13.78
$(f+c)=0.75$	0.75	0.06	0.75	0.07	0.75	0.08	0.74	0.10
$(f+c)=1.00$	1.00	0.08	0.99	0.09	0.99	0.11	0.99	0.13
$(f+c)=1.25$	1.25	0.10	1.24	0.11	1.24	0.14	1.24	0.16

TABLE III

(Continued)

Horizontal Distances and Elevations from Stadia Readings

Minutes	8°		9°		10°		11°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	1.94	13.78	2.45	15.45	3.02	17.10	3.64	18.73
2	1.95	13.84	2.47	15.51	3.04	17.16	3.66	18.78
4	1.97	13.89	2.48	15.56	3.06	17.21	3.68	18.84
6	1.99	13.95	2.50	15.62	3.08	17.26	3.71	18.89
8	2.00	14.01	2.52	15.67	3.10	17.32	3.73	18.95
10	2.02	14.06	2.54	15.73	3.12	17.37	3.75	19.00
12	2.03	14.12	2.56	15.78	3.14	17.43	3.77	19.05
14	2.05	14.17	2.57	15.84	3.16	17.48	3.79	19.11
16	2.07	14.23	2.59	15.89	3.18	17.54	3.82	19.16
18	2.08	14.28	2.61	15.95	3.20	17.59	3.84	19.21
20	2.10	14.34	2.63	16.00	3.22	17.65	3.86	19.27
22	2.12	14.40	2.65	16.06	3.24	17.70	3.88	19.32
24	2.13	14.45	2.67	16.11	3.26	17.76	3.91	19.38
26	2.15	14.51	2.69	16.17	3.28	17.81	3.93	19.43
28	2.17	14.56	2.71	16.22	3.30	17.86	3.95	19.48
30	2.18	14.62	2.72	16.28	3.32	17.92	3.97	19.54
32	2.20	14.67	2.74	16.33	3.34	17.97	4.00	19.59
34	2.22	14.73	2.76	16.39	3.36	18.03	4.02	19.64
36	2.24	14.79	2.78	16.44	3.38	18.08	4.04	19.70
38	2.25	14.84	2.80	16.50	3.40	18.14	4.07	19.75
40	2.27	14.90	2.82	16.55	3.43	18.19	4.09	19.80
42	2.29	14.95	2.84	16.61	3.45	18.24	4.11	19.86
44	2.31	15.01	2.86	16.66	3.47	18.30	4.14	19.91
46	2.32	15.06	2.88	16.72	3.49	18.35	4.16	19.96
48	2.34	15.12	2.90	16.77	3.51	18.41	4.18	20.02
50	2.36	15.17	2.92	16.83	3.53	18.46	4.21	20.07
52	2.38	15.23	2.94	16.88	3.55	18.51	4.23	20.12
54	2.39	15.28	2.96	16.94	3.58	18.57	4.25	20.18
56	2.41	15.34	2.98	16.99	3.60	18.62	4.28	20.23
58	2.43	15.40	3.00	17.05	3.62	18.68	4.30	20.28
60	2.45	15.45	3.02	17.10	3.64	18.73	4.32	20.34
$(f+c)=0.75$	0.74	0.11	0.74	0.12	0.74	0.14	0.73	0.15
$(f+c)=1.00$	0.99	0.15	0.99	0.16	0.98	0.18	0.98	0.20
$(f+c)=1.25$	1.23	0.18	1.23	1.21	1.23	0.23	1.22	0.25

TABLE III
(Continued)

Horizontal Distances and Elevations from Stadia Readings

Minutes	12°		13°		14°		15°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	4.32	20.34	5.06	21.92	5.85	23.47	6.70	25.00
2	4.35	20.39	5.09	21.97	5.88	23.52	6.73	25.05
4	4.37	20.44	5.11	22.02	5.91	23.58	6.76	25.10
6	4.39	20.50	5.14	22.08	5.93	23.63	6.79	25.15
8	4.42	20.55	5.16	22.13	5.96	23.68	6.82	25.20
10	4.44	20.60	5.19	22.18	5.99	23.73	6.84	25.25
12	4.47	20.66	5.21	22.23	6.02	23.78	6.87	25.30
14	4.49	20.71	5.24	22.28	6.05	23.83	6.90	25.35
16	4.51	20.76	5.27	22.34	6.07	23.88	6.93	25.40
18	4.54	20.81	5.29	22.39	6.10	23.93	6.96	25.45
20	4.56	20.87	5.32	22.44	6.13	23.99	6.99	25.50
22	4.59	20.92	5.34	22.49	6.16	24.04	7.02	25.55
24	4.61	20.97	5.37	22.54	6.19	24.09	7.05	25.60
26	4.64	21.03	5.40	22.60	6.21	24.14	7.08	25.65
28	4.66	21.08	5.42	22.65	6.24	24.19	7.11	25.70
30	4.68	21.13	5.45	22.70	6.27	24.24	7.14	25.75
32	4.71	21.18	5.48	22.75	6.30	24.29	7.17	25.80
34	4.73	21.24	5.50	22.80	6.33	24.34	7.20	25.85
36	4.76	21.29	5.53	22.85	6.35	24.39	7.23	25.90
38	4.78	21.34	5.56	22.91	6.38	24.44	7.26	25.95
40	4.81	21.39	5.58	22.96	6.41	24.49	7.29	26.00
42	4.83	21.45	5.61	23.01	6.44	24.55	7.32	26.05
44	4.86	21.50	5.64	23.06	6.47	24.60	7.35	26.10
46	4.88	21.55	5.66	23.11	6.50	24.65	7.38	26.15
48	4.91	21.60	5.69	23.16	6.53	24.70	7.41	26.20
50	4.93	21.66	5.72	23.22	6.55	24.75	7.44	26.25
52	4.96	21.71	5.74	23.27	6.58	24.80	7.47	26.30
54	4.98	21.76	5.77	23.32	6.61	24.85	7.51	26.35
56	5.01	21.81	5.80	23.37	6.64	24.90	7.54	26.40
58	5.03	21.87	5.83	23.42	6.67	24.95	7.57	26.45
60	5.06	21.92	5.85	23.47	6.70	25.00	7.60	26.50
$(f+c)=0.75$	0.73	0.16	0.73	0.17	0.73	0.19	0.72	0.20
$(f+c)=1.00$	0.98	0.22	0.97	0.23	0.97	0.25	0.96	0.27
$(f+c)=1.25$	1.22	0.27	1.21	0.29	1.21	0.31	1.20	0.34

TABLE III

(Continued)

Horizontal Distances and Elevations from Stadia Readings

Minutes	16°		17°		18°		19°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev	Hor. Corr.	Diff. Elev.
0	7.60	26.50	8.55	27.96	9.55	29.39	10.60	30.78
2	7.63	26.55	8.58	28.01	9.58	29.44	10.64	30.83
4	7.66	26.59	8.61	28.06	9.62	29.48	10.67	30.87
6	7.69	26.64	8.65	28.10	9.65	29.53	10.71	30.92
8	7.72	26.69	8.68	28.15	9.69	29.58	10.74	30.97
10	7.75	26.74	8.71	28.20	9.72	29.62	10.78	31.01
12	7.78	26.79	8.74	28.25	9.76	29.67	10.82	31.06
14	7.81	26.84	8.78	28.30	9.79	29.72	10.85	31.10
16	7.85	26.89	8.81	28.34	9.82	29.76	10.89	31.15
18	7.88	26.94	8.84	28.39	9.86	29.81	10.92	31.19
20	7.91	26.99	8.88	28.44	9.89	29.86	10.96	31.24
22	7.94	27.04	8.91	28.49	9.93	29.90	11.00	31.28
24	7.97	27.09	8.94	28.54	9.96	29.95	11.04	31.33
26	8.00	27.13	8.98	28.58	10.00	30.00	11.07	31.38
28	8.03	27.18	9.01	28.63	10.03	30.04	11.11	31.42
30	8.07	27.23	9.04	28.68	10.07	30.09	11.14	31.47
32	8.10	27.28	9.08	28.73	10.10	30.14	11.18	31.51
34	8.13	27.33	9.11	28.77	10.14	30.19	11.22	31.56
36	8.16	27.38	9.14	28.82	10.17	30.23	11.25	31.60
38	8.19	27.43	9.18	28.87	10.21	30.28	11.29	31.65
40	8.23	27.48	9.21	28.92	10.24	30.32	11.33	31.69
42	8.26	27.52	9.24	28.96	10.28	30.37	11.36	31.74
44	8.29	27.57	9.28	29.01	10.31	30.41	11.40	31.78
46	8.32	27.62	9.31	29.06	10.35	30.46	11.44	31.83
48	8.35	27.67	9.34	29.11	10.39	30.51	11.47	31.87
50	8.39	27.72	9.38	29.15	10.42	30.55	11.51	31.92
52	8.42	27.77	9.41	29.20	10.46	30.60	11.55	31.96
54	8.45	27.81	9.45	29.25	10.49	30.65	11.59	32.01
56	8.48	27.86	9.48	29.30	10.53	30.69	11.62	32.05
58	8.52	27.91	9.52	29.34	10.56	30.74	11.66	32.09
60	8.55	27.96	9.55	29.39	10.60	30.78	11.70	32.14
$(f+c)=0.75$	0.72	0.21	0.72	0.23	0.71	0.24	0.71	0.25
$(f+c)=1.00$	0.86	0.28	0.95	0.30	0.95	0.32	0.94	0.33
$(f+c)=1.25$	1.20	0.35	1.19	0.38	1.19	0.40	1.18	0.42

TABLE III
(Continued)

Horizontal Distances and Elevations from Stadia Readings

Minutes	20°		21°		22°		23°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	11.70	32.14	12.84	33.46	14.03	34.73	15.27	35.97
2	11.74	32.18	12.88	33.50	14.07	34.77	15.31	36.01
4	11.77	32.23	12.92	33.54	14.11	34.82	15.35	36.05
6	11.81	32.27	12.96	33.59	14.15	34.86	15.39	36.09
8	11.85	32.32	13.00	33.63	14.20	34.90	15.43	36.13
10	11.89	32.36	13.04	33.67	14.24	34.94	15.48	36.17
12	11.92	32.41	13.08	33.72	14.28	34.98	15.52	36.21
14	11.96	32.45	13.12	33.76	14.32	35.02	15.56	36.25
16	12.00	32.49	13.16	33.80	14.36	35.07	15.60	36.29
18	12.04	32.54	13.20	33.84	14.40	35.11	15.65	36.33
20	12.07	32.58	13.23	33.89	14.44	35.15	15.69	36.37
22	12.11	32.63	13.27	33.93	14.48	35.19	15.73	36.41
24	12.15	32.67	13.31	33.97	14.52	35.23	15.77	36.45
26	12.19	32.72	13.35	34.01	14.56	35.27	15.82	36.49
28	12.23	32.76	13.39	34.06	14.60	35.31	15.86	36.53
30	12.26	32.80	13.43	34.10	14.64	35.36	15.90	36.57
32	12.30	32.85	13.47	34.14	14.69	35.40	15.94	36.61
34	12.34	32.89	13.51	34.18	14.73	35.44	15.99	36.65
36	12.38	32.93	13.55	34.23	14.77	35.48	16.03	36.69
38	12.42	32.98	13.59	34.27	14.81	35.52	16.07	36.73
40	12.46	33.02	13.63	34.31	14.85	35.56	16.11	36.77
42	12.49	33.07	13.67	34.35	14.89	35.60	16.16	36.80
44	12.53	33.11	13.71	34.40	14.93	35.64	16.20	36.84
46	12.57	33.15	13.75	34.44	14.98	35.68	16.24	36.88
48	12.61	33.20	13.79	34.48	15.02	35.72	16.28	36.92
50	12.65	33.24	13.83	34.52	15.06	35.76	16.33	36.96
52	12.69	33.28	13.87	34.57	15.10	35.80	16.37	37.00
54	12.73	33.33	13.91	34.61	15.14	35.85	16.41	37.04
56	12.76	33.37	13.95	34.65	15.18	35.89	16.46	37.08
58	12.80	33.41	13.99	34.69	15.23	35.93	16.50	37.12
60	12.84	33.46	14.03	34.73	15.27	35.97	16.54	37.16
$(f+c)=0.75$	0.70	0.26	0.70	0.27	0.69	0.29	0.69	0.30
$(f+c)=1.00$	0.94	0.35	0.93	0.37	0.92	0.38	0.92	0.40
$(f+c)=1.25$	1.17	0.44	1.16	0.46	1.15	0.48	1.15	0.50

TABLE III

(Continued)

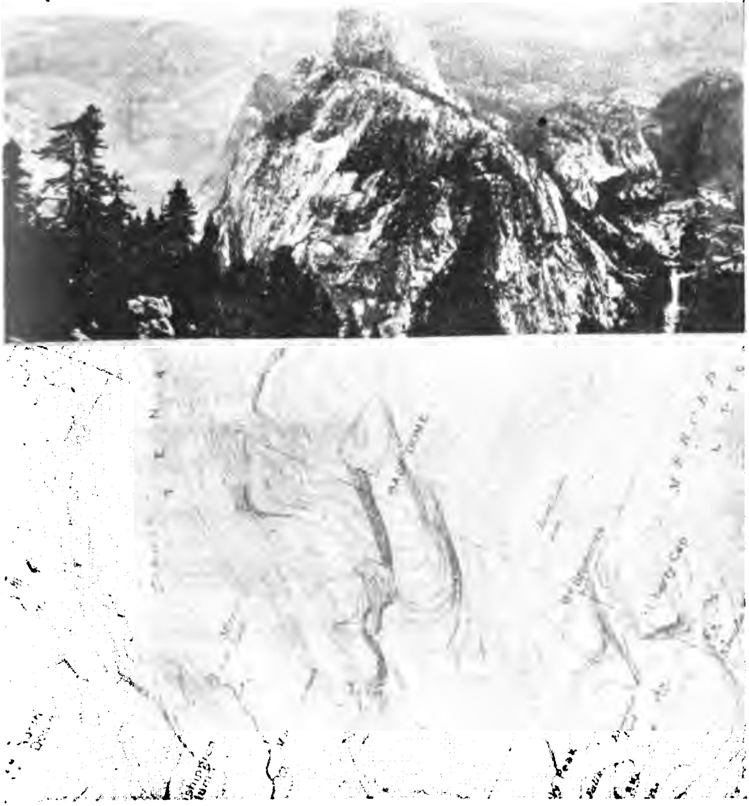
Horizontal Distances and Elevations from Stadia Readings

Minutes	24°		25°		26°		27°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	16.54	37.16	17.86	38.30	19.22	39.40	20.61	40.45
2	16.59	37.20	17.91	38.34	19.26	39.44	20.66	40.49
4	16.63	37.23	17.95	38.38	19.31	39.47	20.70	40.52
6	16.67	37.27	17.99	38.41	19.35	39.51	20.75	40.55
8	16.72	37.31	18.04	38.45	19.40	39.54	20.80	40.59
10	16.76	37.35	18.08	38.49	19.45	39.58	20.85	40.62
12	16.80	37.39	18.13	38.53	19.49	39.61	20.89	40.66
14	16.85	37.43	18.17	38.56	19.54	39.65	20.94	40.69
16	16.89	37.47	18.22	38.60	19.59	39.69	20.99	40.72
18	16.93	37.51	18.26	38.64	19.63	39.72	21.04	40.76
20	16.98	37.54	18.31	38.67	19.68	39.76	21.08	40.79
22	17.02	37.58	18.35	38.71	19.72	39.79	21.13	40.82
24	17.07	37.62	18.40	38.75	19.77	39.83	21.18	40.86
26	17.11	37.66	18.44	38.78	19.82	39.86	21.23	40.89
28	17.15	37.70	18.49	38.82	19.86	39.90	21.27	40.92
30	17.20	37.74	18.53	38.86	19.91	39.93	21.32	40.96
32	17.24	37.77	18.58	38.89	19.96	39.97	21.37	40.99
34	17.28	37.81	18.62	38.93	20.00	40.00	21.42	41.02
36	17.33	37.85	18.67	38.97	20.05	40.04	21.46	41.06
38	17.37	37.89	18.72	39.00	20.10	40.07	21.51	41.09
40	17.42	37.93	18.76	39.04	20.14	40.11	21.56	41.12
42	17.46	37.96	18.81	39.08	20.19	40.14	21.62	41.16
44	17.51	38.00	18.85	39.11	20.24	40.18	21.66	41.19
46	17.55	38.04	18.90	39.15	20.28	40.21	21.70	41.22
48	17.59	38.08	18.94	39.18	20.33	40.24	21.75	41.26
50	17.64	38.11	18.99	39.22	20.38	40.28	21.80	41.29
52	17.68	38.15	19.03	29.26	20.42	40.31	21.85	41.32
54	17.73	38.19	19.08	39.29	20.47	40.35	21.90	41.35
56	17.77	38.23	19.13	39.33	20.52	40.38	21.94	41.39
58	17.82	38.26	19.17	39.36	20.56	40.42	21.99	41.42
60	17.86	38.30	19.22	39.40	20.61	40.45	22.04	41.45
$(f+c)=0.75$	0.68	0.31	0.68	0.32	0.67	0.33	0.66	0.35
$(f+c)=1.00$	0.91	0.41	0.90	0.43	0.89	0.45	0.89	0.46
$(f+c)=1.25$	1.14	0.52	1.13	0.54	1.12	0.56	1.11	0.58

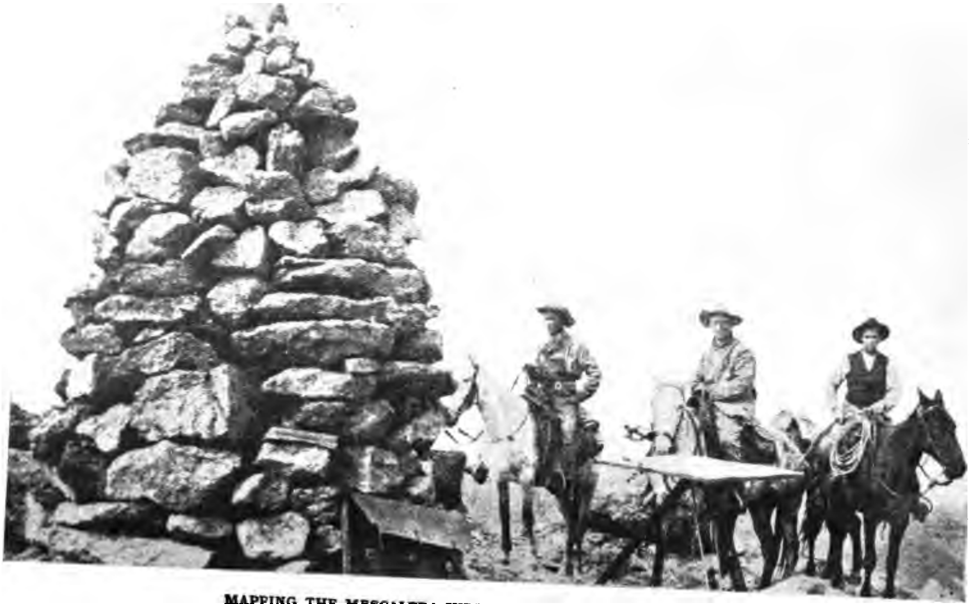
TABLE III
(Continued)

Horizontal Distances and Elevations from Stadia Readings

Minutes	28°		29°		30°	
	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.	Hor. Corr.	Diff. Elev.
0	22.04	41.45	23.50	42.40	25.00	43.30
2	22.09	41.48	23.55	42.43	25.05	43.33
4	22.14	41.52	23.60	42.46	25.10	43.36
6	22.19	41.55	23.65	42.49	25.15	43.39
8	22.23	41.58	23.70	42.53	25.20	43.42
10	22.28	41.61	23.75	42.56	25.25	43.45
12	22.33	41.65	23.80	42.59	25.30	43.47
14	22.38	41.68	23.85	42.62	25.35	43.50
16	22.43	41.71	23.90	42.65	25.40	43.53
18	22.48	41.74	23.95	42.68	25.45	43.56
20	22.52	41.77	24.00	42.71	25.51	43.59
22	22.58	41.81	24.05	42.74	25.56	43.62
24	22.62	41.84	24.10	42.77	25.61	43.65
26	22.67	41.87	24.15	42.80	25.66	43.67
28	22.72	41.90	24.20	42.83	25.71	43.70
30	22.77	41.93	24.25	42.86	25.76	43.73
32	22.82	41.97	24.30	42.89	25.81	43.76
34	22.87	42.00	24.35	42.92	25.86	43.79
36	22.91	42.03	24.40	42.95	25.91	43.82
38	22.96	42.06	24.45	42.98	25.96	43.84
40	23.01	42.09	24.50	43.01	26.01	43.87
42	23.06	42.12	24.55	43.04	26.07	43.90
44	23.11	42.15	24.60	43.07	26.12	43.93
46	23.16	42.19	24.65	43.10	26.17	43.95
48	23.21	42.22	24.70	43.13	26.22	43.98
50	23.26	42.25	24.75	43.16	26.27	44.01
52	23.31	42.28	24.80	43.18	26.32	44.04
54	23.36	42.31	24.85	43.21	26.37	44.07
56	23.41	42.34	24.90	43.24	26.42	44.09
58	23.45	42.37	24.95	43.27	26.48	44.12
60	23.50	42.40	25.00	43.30	26.53	44.15
$(f+c)=0.75$	0.66	0.36	0.65	0.37	0.65	0.38
$(f+c)=1.00$	0.88	0.48	0.87	0.49	0.86	0.51
$(f+c)=1.25$	1.10	0.60	1.09	0.62	1.08	0.64



PHOTOGRAPHIC VIEW AND CONTOUR MAP IN YOSEMITE VALLEY, CALIFORNIA
These two reproductions of the same area give a good idea of the two methods of showing topographical features.
Courtesy of the United States Geological Survey, Washington, D. C.



MAPPING THE MESCALERA INDIAN RESERVATION, NEW MEXICO
Triangulation signal and party on Sierra Blanca, 12,000 feet above sea level.
Courtesy of United States Geological Survey, Washington, D. C.

PLANE SURVEYING

PART III

LAND AND CONSTRUCTION SURVEYING

DETERMINATION OF MERIDIAN

Use of True Meridian. It is especially important in land surveys that the bearings of the boundaries be referred to a true meridian. In order to determine the declination for use with a compass, many surveyors lay out a true meridian, marking the ends by stakes, and find the declination by placing a compass on this line and reading its bearing, which equals the declination. In other surveys it is desirable also to have a true north direction.

Two general methods are available for determining a true meridian, viz, by observation on a star; and by observation on the sun. If the star method is carefully used, it will give results within 15 or 20 seconds, but it requires taking an observation at night, whereas, by the sun method, observations may be taken almost any time during the day. The results in the latter case, however, are generally less accurate and seldom can be expected to be closer than 1 minute.

Before describing either of these methods, it will be necessary to explain certain astronomical terms. Methods will also be given for determining latitude, as the value of the latitude is required in the computations by either method.

Definition of Astronomical Relations. *Celestial Sphere.* The sun, planets, and stars appear to be moving in their paths as though these paths were on the interior of a huge hollow globe, or sphere, with the earth at its center. This sphere is called the *celestial sphere*, and for all practical purposes its radius may be taken as infinite.

In Fig. 88 is shown the celestial sphere with the earth at its center, which will serve to make these definitions clear. It should be noted that the earth is shown many times too large; the radius of the sphere being infinite, the earth should be represented by a point at the center.

Celestial Poles. The points in the celestial sphere where the earth's axis prolonged would cut it are the celestial poles. There are two, the north pole (N) and the south pole (S).

Zenith. The point in the celestial sphere directly over the observer's head, or the point where the prolongation of a plumb line would cut the celestial sphere, is the zenith.

Celestial Equator. The great circle in which a plane passing through the center of the earth perpendicular to its axis, that is, through the earth's equator, would cut the celestial sphere is the celestial equator.

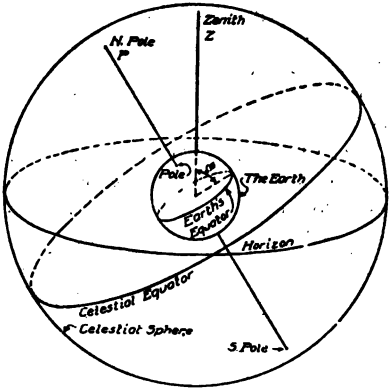


Fig. 88. Diagram of Celestial Sphere

Horizon. The horizon is the great circle in which a plane—horizontal—which is perpendicular to a plumb line at the point of observation would cut the celestial sphere.

Latitude. The angle at the center of the earth between a vertical line through the point of observation and the plane of the equator is the latitude indicated by the Greek letter *phi* (ϕ).

Declination. The angle at the center of the earth between the line to the center of the sun and the plane of the equator is the declination of the sun, indicated by the Greek letter *delta* (δ), Fig. 93. This angle is changing all the time, owing to the fact that the earth's axis is inclined and not perpendicular to its orbit, or path about the sun. In the winter the sun gets as far south of the equator as 23° 27′; in the middle of March, at the vernal equinox, its declination is 0; in June it reaches its greatest northern, or plus (+), declination of about 23° 27′, decreases again to 0 in September, at the autumn equinox, and passes below the equator. The value of the declination at any time may be found from a *Solar Ephemeris*, such as is published by many instrument makers and usually is sent free on request.

METHODS OF DETERMINING LATITUDE

Use of Maps. In many cases the latitude of a place may be found from maps. By means of such topographic maps as are published by the U. S. Geological Survey, it is easy to scale off the latitude of any point very accurately.

Observation on Star for Latitude

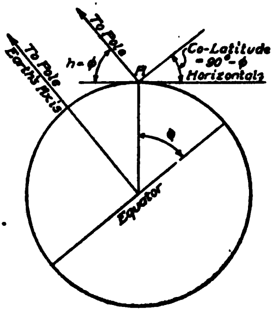


Fig. 89. Diagram of Latitude Determination by Observation on Star

Principle of Determination. Referring to Fig. 89, it is easily seen that the altitude *h*, or vertical angle of a point above the horizontal, of the celestial pole will equal the latitude of the point of observation. This is true of any point of observation, such as *A*, because the direction of the celestial pole as seen from *A* will be parallel to the earth's axis, since the pole is an infinite distance from the earth.

If, therefore, we know where to look for the pole in the sky, we could simply direct our transit at this point and measure the vertical angle, which would be the latitude. Unfortunately, there is no star exactly at the pole. The nearest bright star to the pole is the polestar, or Polaris. Any northern star could be used, but Polaris is the one usually selected for observations in the northern hemisphere; in the southern hemisphere one of the stars in the Southern Cross is frequently used. Polaris is very near the pole. Its declination changes slightly and is about 88° 50′; hence the angle at the earth between the pole and Polaris, or 90° minus the declination, or the pole distance, is about 1° 10′. (See Table IV.)

Owing to the rotation of the earth from west to east, the celestial sphere and the stars appear to revolve from east to west, and Polaris, therefore, moves in a small circle of 1 degree 10 minutes radius about the pole, moving in a counterlockwise direction

TABLE IV

Culminations of Polaris

(For 1916 only—See Note, Bottom p. 140)

Date	Upper Culminations Local Mean Time	Lower Culminations Local Mean Time	Difference for 1 Day	Polar Distance
	h. m.	h. m.	m.	(1°7′)
Jan. 1	6 48.5 P.M.	6 50.4 A.M.	3.95	68″
Jan. 16	5 49.2 P.M.	5 51.1 A.M.	3.95	67″
Feb. 1	4 46.1 P.M.	4 48.0 A.M.	3.95	67″
Feb. 16	3 46.8 P.M.	3 48.9 A.M.	3.95	69″
Mar. 1	2 51.6 P.M.	2 53.5 A.M.	3.94	71″
Mar. 16	1 52.5 P.M.	1 54.5 A.M.	3.94	75″
Apr. 1	0 49.5 P.M.	0 51.5 A.M.	3.93	80″
Apr. 16	11 50.5 A.M.	11 48.5 P.M.	3.93	84″
May 1	10 51.6 A.M.	10 49.6 P.M.	3.92	89″
May 16	9 52.8 A.M.	9 50.8 P.M.	3.92	93″
June 1	8 50.1 A.M.	8 48.1 P.M.	3.92	96″
June 16	7 51.4 A.M.	7 49.5 P.M.	3.91	98″
July 1	6 52.7 A.M.	6 50.7 P.M.	3.91	98″
July 16	5 54.0 A.M.	6 52.1 P.M.	3.91	98″
Aug. 1	4 51.4 A.M.	4 49.5 P.M.	3.91	96″
Aug. 16	3 52.6 A.M.	Daylight; Polaris invisible at 45°	3.92	93″
Sept. 1	2 50.0 A.M.		3.92	89″
Sept. 16	1 51.2 A.M.		3.92	84″
Oct. 1	0 52.4 A.M.		3.93	78″
Oct. 16	11 49.5 P.M.		3.93	72″
Nov. 1	10 46.6 P.M.		3.93	66″
Nov. 16	9 47.6 P.M.		3.94	61″
Dec. 1	8 48.4 P.M.		3.94	56″
Dec. 16	7 49.3 P.M.		3.94	52″

and completing its circle once in a day. When, as diagrammatically shown in Fig. 90, Polaris is directly above the pole, it is said to be at upper *culmination;* when directly east of the pole, it is at eastern *elongation,* etc. The culminations of Polaris are given in Table IV and the times of elongation are as given in Table V; if we observe the vertical angle of Polaris at either elongation, it will equal ϕ. This cannot be observed very accurately, however, as Polaris is moving rapidly upward at eastern elongation and downward at western elongation, and a slight error in the time at which the observation was taken would make a big error in the vertical angle, or ϕ. For this reason, we observe the

TABLE V

Elongations of Polaris

(For 1916 only—See Note, Bottom p. 140)

Date	Elongation	Local Mean Time	Difference for 1 Day	Date	Elongation	Local Mean Time	Difference for 1 Day
		h. m.	m.			h. m.	m.
Jan. 1	W	0 46.9 A.M.	3.95	July 1	E	0 58.2 A.M.	3.91
Jan. 16	W	11 43.7 P.M.	3.95	July 16	E	11 55.6 P.M.	3.91
Feb. 1	W	10 40.6 P.M.	3.95	Aug. 1	E	10 53.0 P.M.	3.91
Feb. 16	W	9 41.3 P.M.	3.95	Aug. 16	E	9 54.2 P.M.	3.92
Mar. 1	W	8 46.1 P.M.	3.94	Sept. 1	E	8 51.4 P.M.	3.92
Mar. 16	W	7 47.0 P.M.	3.94	Sept. 16	E	7 53.0 P.M.	3.92
Apr. 1	W	6 44.0 P.M.	3.93	Oct. 1	E	6 53.9 P.M.	3.93
Apr. 16	W	5 45.0 P.M.	3.93	Oct. 16	E	5 55.0 P.M.	3.93
May 1	E	4 57.2 A.M.	3.92	Nov. 1	W	4 45.0 A.M.	3.93
May 16	E	3 58.3 A.M.	3.92	Nov. 16	W	3 46.0 A.M.	3.94
June 1	E	2 55.6 A.M.	3.92	Dec. 1	W	2 46.9 A.M.	3.94
June 16	E	1 56.8 A.M.	3.91	Dec. 16	W	1 47.7 A.M.	3.94

star at either the upper or lower culmination, following it with our instrument until it reaches its highest or lowest point, as the case may be, and correcting the observed angle by the pole distance. It should be noted that Polaris is invisible when at culmination during certain times of the year, as indicated in Table IV, and accordingly some other method must be used at that period.

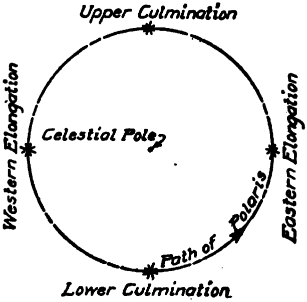


Fig. 90. Orbit of Polaris about Celestial Pole

Method of Observation on Polaris for Altitude. The usual observation is made as follows: Suppose the latitude is to be determined for September 11, 1916. From Table IV it is seen that we have to observe the upper culmination, which occurs at about 2:50 A.M. minus the correction for 10 days—10 times the difference for 1 day equals 39.2 minutes—or 2:10.8 A.M. About one-half hour beforehand, set up the instrument at any convenient point and carefully level it up.

TABLE VI*

Azimuth of Polaris at Elongation

(1916)

Latitude	Azimuth			Latitude	Azimuth			Latitude	Azimuth			Latitude	Azimuth		
(°)	(°)	(′)	(″)	(°)	(°)	(′)	(″)	(°)	(°)	(′)	(″)	(°)	(°)	(′)	(″)
10	1	9	23	25	1	15	24	40	1	29	12	55	1	59	9
11	1	9	37	26	1	16	2	41	1	30	33	56	2	2	13
12	1	9	52	27	1	16	42	42	1	31	57	57	2	5	29
13	1	10	8	28	1	17	24	43	1	33	26	58	2	8	58
14	1	10	26	29	1	18	8	44	1	35	0	59	2	12	42
15	1	10	45	30	1	18	54	45	1	36	39	60	2	16	42
16	1	11	5	31	1	19	43	46	1	38	23	61	2	20	59
17	1	11	27	32	1	20	35	47	1	40	12	62	2	25	35
18	1	11	51	33	1	21	29	48	1	42	8	63	2	30	33
19	1	12	16	34	1	22	26	49	1	44	10	64	2	35	55
20	1	12	43	35	1	23	25	50	1	46	19	65	2	41	44
21	1	13	12	36	1	24	28	51	1	48	36	66	2	48	4
22	1	13	42	37	1	25	34	52	1	51	0	67	2	54	57
23	1	14	14	38	1	26	43	53	1	53	34	68	3	2	29
24	1	14	48	39	1	27	56	54	1	56	16	69	3	10	46

* Table VI has been computed for April 1 and September 26, 1916 (polar distance = 1°8′20″). For other dates, the correction from the appended table should be applied to the azimuth taken from Table VI.

Azimuth Corrections for Table VI

Date	Latitude			Date	Latitude		
	30°	40°	50°		30°	40°	50°
	Correction				Correction		
	(″)	(″)	(″)		(″)	(″)	(″)
Jan. 1	−15	−17	−20	July 1	+20	+22	+26
Feb. 1	−16	−18	−22	Aug. 1	+17	+20	+23
Mar. 1	−10	−12	−14	Sept. 1	+ 9	+10	+12
Apr. 1	− 1	− 1	− 2	Oct. 1	− 3	− 4	− 5
May 1	+ 9	+10	+12	Nov. 1	−17	−20	−23
June 1	+17	+20	+23	Dec. 1	−30	−33	−39

NOTE: Tables IV, V, and VI have been taken from the *Solar Ephemeris* for 1916, which is published yearly by the Eugene Dietzgen Company, Chicago. Students should secure a copy of this pamphlet, which may be obtained by addressing the publishers. Tables IV, V, and VI are for 1916, only.

The adjustment should have been tested previously, particularly the long bubble and the vertical arc. Direct the instrument at Polaris and carefully set the horizontal hair on the star. If not already known to the observer, Polaris may be found in the sky by means of the Pointers, which are two stars in the constellation of Ursa Major (known as the Big Dipper). A line drawn through these stars passes very near the pole-star, as shown in Fig. 91.

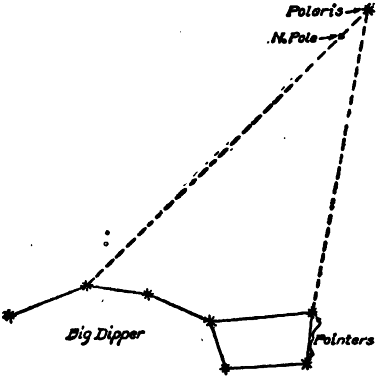


Fig. 91. Diagram Showing Position of Polaris with Reference to Big Dipper

The student will have considerable difficulty at first in seeing and locating the star through the telescope. He should remember that the instrument must be focused, and he may get the approximate direction of the star by placing his instrument in the meridian by means of the compass and by elevating the telescope to about the supposed latitude. Also, it will be impossible to see the cross hairs without illumination, as they do not show against the dark sky, so a small piece of thin white cardboard cut as shown in Fig. 92(*a*) should be provided. If this device is placed inside the sunshade of the transit telescope and the flap bent over at about 45 degrees, Fig. 92(*b*), it will be possible to see Polaris through the telescope and the hole in the flap, while light from a lantern held to one side will be reflected down the telescope tube by the flap, thus illuminating the cross hairs.

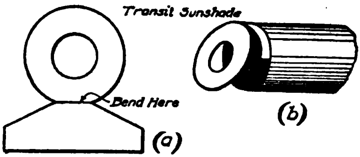


Fig. 92. Deflector for Illumination of Cross Hairs

Polaris should be followed, moving the horizontal hair upward, so as to keep it on the star, until it reaches its highest point, when it will start to move downward. The vertical angle for the

TABLE VII

Mean Refraction at Various Altitudes

(Barometer, 30 Inches; Fahrenheit Thermometer, 50°)

Altitude	Refraction		Altitude	Refraction	
(°)	(′)	(″)	(°)	(′)	(″)
10	5	19	20	2	39
11	4	51	25	2	04
12	4	27	30	1	41
13	4	07	35	1	23
14	3	49	40	1	09
15	3	34	45	0	58
16	3	20	50	0	49
17	3	08	60	0	34
18	2	57	70	0	21
19	2	48	80	0	10

highest position is then read and, if the transit has a complete vertical arc, the telescope should be reversed and the angle read again. Taking an average of the two readings eliminates the errors in setting the vertical arc, etc.

Deduction of Latitude. To get the latitude, simply subtract from the average vertical angle the polar distance as given in Table IV. If the lower culmination is observed, then add the pole distance.

Allowance for Refraction. It should also be noted that the observed vertical angle should be corrected to allow for refraction. The earth's atmosphere *refracts*, or bends, the rays of light passing through it, just as water does, as illustrated by a straight stick which appears to be bent when one end is placed in the water. Refraction always makes objects appear higher than they really are; hence it must be subtracted from the observed altitude, or vertical angle, to get the true angle. The value may be taken by interpolation from Table VII.

Example. In the determination of latitude by means of observations, on Polaris, September 11, 1916, as discussed in the preceding pages, the observed vertical angle was 38° 20′ 30″. Find the latitude.

Ans. 37° 10′ 48″

Observation on Sun for Latitude

Principle of Latitude Determination. The principle involved in this case is illustrated in Fig. 93 and is similar to that given above. Thus it is seen that

$$90° - h = \phi - \delta$$

hence

$$\phi = 90° - h + \delta$$

in which *h* is the altitude of the sun, to be determined by observation, and δ is the declination, to be taken from the *Solar Ephemeris*. Note that δ is minus (−) in winter and must be subtracted. This relation only holds when the sun is on the meridian, or at noon; hence the observation is made as follows:

Altitude of Sun. A short time before noon the transit is set up at a convenient point, carefully leveled, and the telescope directed at the sun. It will, of course, be impossible to look at the sun through the telescope unless a colored glass, such as is furnished with some instruments, is used to protect the eyes. In case no glass is at hand, a piece of white cardboard is held 2 or 3 inches back of the eyepiece. The sun will appear on this paper as a bright white circle and, by properly adjusting the eyepiece and focusing the objective, the cross hairs will appear as fine black lines crossing the image of the sun. The top or bottom of the sun is brought tangent to the horizontal wire by means of the vertical motion, and the sun is followed in this way until it reaches its highest altitude, which will occur at about noon.

Correction of Altitude. The maximum altitude must then be corrected for refraction by Table VII, and for the error in setting the vertical arc, if there is any. Note that an instrument with a complete vertical arc is preferable, as the telescope can be reversed and a second observation taken, as in the observation on Polaris. Sixteen minutes must be added to or subtracted from the corrected altitude, according to whether the lower or upper edge, or limb, of the sun was observed. This is the semidiameter of the sun and gives the true altitude of the center of the sun.

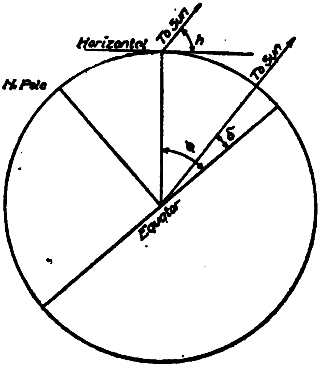
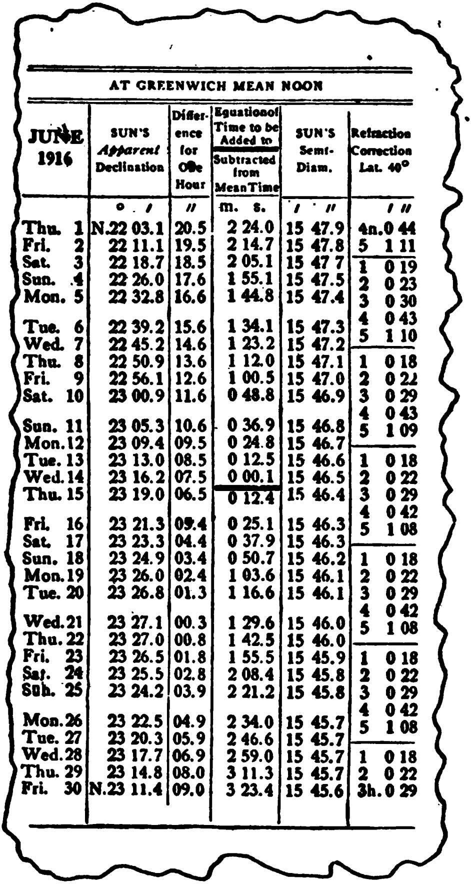


Fig. 93. Diagram of Latitude Determination by Observation on Sun

Declination of Sun. The declination of the sun is obtained from the *Solar Ephemeris*, a sample page of which is shown in Table VIII. Note that the declination is given for Greenwich, England,

TABLE VIII

Sample Page from "Solar Ephemeris" for 1916*



AT GREENWICH MEAN NOON

JUNE 1916	SUN'S *Apparent* Declination	Difference for One Hour	Equation of Time to be Added to Subtracted from Mean Time	SUN'S Semi-Diam.	Refraction Correction Lat. 40°
	° ′	″	m. s.	′ ″	′ ″
Thu. 1	N.22 03.1	20.5	2 24.0	15 47.9	4n.0 44
Fri. 2	22 11.1	19.5	2 14.7	15 47.8	5 1 11
Sat. 3	22 18.7	18.5	2 05.1	15 47 7	1 0 19
Sun. 4	22 26.0	17.6	1 55.1	15 47.5	2 0 23
Mon. 5	22 32.8	16.6	1 44.8	15 47.4	3 0 30
					4 0 43
Tue. 6	22 39.2	15.6	1 34.1	15 47.3	5 1 10
Wed. 7	22 45.2	14.6	1 23.2	15 47.2	
Thu. 8	22 50.9	13.6	1 12.0	15 47.1	1 0 18
Fri. 9	22 56.1	12.6	1 00.5	15 47.0	2 0 22
Sat. 10	23 00.9	11.6	0 48.8	15 46.9	3 0 29
					4 0 43
Sun. 11	23 05.3	10.6	0 36.9	15 46.8	5 1 09
Mon. 12	23 09.4	09.5	0 24.8	15 46.7	
Tue. 13	23 13.0	08.5	0 12.5	15 46.6	1 0 18
Wed. 14	23 16.2	07.5	0 00.1	15 46.5	2 0 22
Thu. 15	23 19.0	06.5	0 12.4	15 46.4	3 0 29
					4 0 42
Fri. 16	23 21.3	05.4	0 25.1	15 46.3	5 1 08
Sat. 17	23 23.3	04.4	0 37.9	15 46.3	
Sun. 18	23 24.9	03.4	0 50.7	15 46.2	1 0 18
Mon. 19	23 26.0	02.4	1 03.6	15 46.1	2 0 22
Tue. 20	23 26.8	01.3	1 16.6	15 46.1	3 0 29
					4 0 42
Wed. 21	23 27.1	00.3	1 29.6	15 46.0	5 1 08
Thu. 22	23 27.0	00.8	1 42.5	15 46.0	
Fri. 23	23 26.5	01.8	1 55.5	15 45.9	1 0 18
Sat. 24	23 25.5	02.8	2 08.4	15 45.8	2 0 22
Sun. 25	23 24.2	03.9	2 21.2	15 45.8	3 0 29
					4 0 42
Mon. 26	23 22.5	04.9	2 34.0	15 45.7	5 1 08
Tue. 27	23 20.3	05.9	2 46.6	15 45.7	
Wed. 28	23 17.7	06.9	2 59.0	15 45.7	1 0 18
Thu. 29	23 14.8	08.0	3 11.3	15 45.7	2 0 22
Fri. 30	N.23 11.4	09.0	3 23.4	15 45.6	3h.0 29

noon, which corresponds to 7 A.M. for the 75°-meridian, or eastern time belt. Thus the declination in New York City, June 12, 1916, at noon equaled 23° 9′ 21.8″+(9.53″×5h), or 23° 10′ 10″.

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*See footnote, p. 140.

Example. Suppose an observation taken on the sun's lower limb for latitude; place located in central time belt; observed altitude 41° 21′ 30″; date, June 24, 1916. Find the latitude.

Ans. $90^\circ-[(41^\circ\ 21'\ 30''-1'\ 06''+16')-(23^\circ\ 25'\ 32.7''-6\times 2.82'')]=71^\circ\ 48'\ 52''$

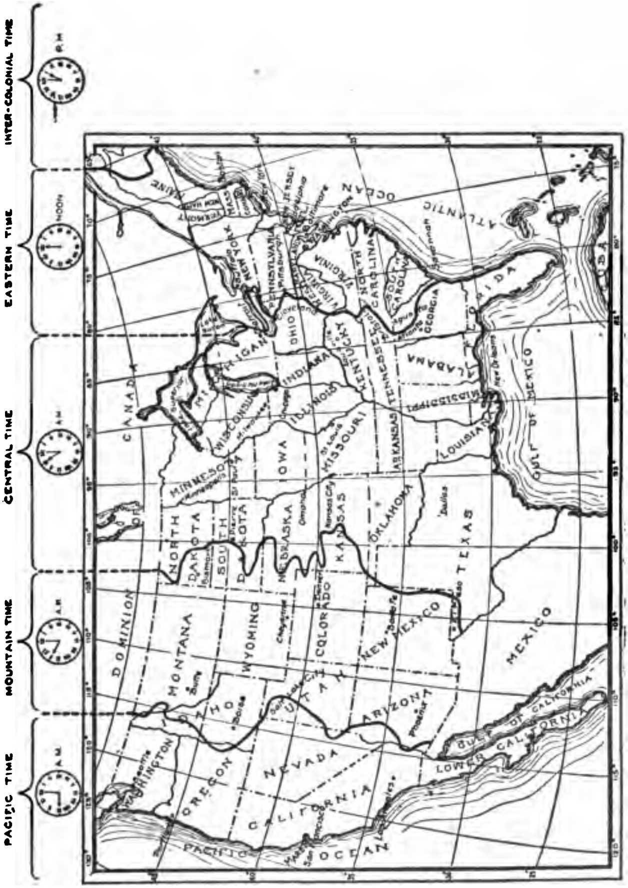

Fig. 94. Map of United States Showing Time Zones

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* Watches are set to keep standard time. There are four time belts in the United States: eastern (75°-meridian); central (90°-meridian); mountain (105°); and western (120°), Fig. 94. Twelve, noon, at Greenwich corresponds to 7 A.M., 6 A.M., 5 A.M., and 4 A.M., respectively, in these different time belts, that is, to obtain standard time, consult the time belts on the map and allow 15° per hour from Greenwich; for example, Denver, Colorado, is mountain belt, Meridian 105°, and therefore its time is 105 ÷ 15, or 7 hours earlier than Greenwich.

METHODS OF DETERMINING MERIDIAN

Observation on Polaris

Principle of Determination. If Polaris were observed at culmination, that is, when it is directly above or below the pole, Fig. 90, all that would be necessary would be to sight on the star at the time of culmination, then to plunge the telescope and to set a point vertically under the star on the ground. Such a line would be a true meridian. The difficulty with this procedure, however, is that Polaris is moving rapidly east or west at culminations, and a slight error in the time of observation would make a big error in the direction observed. For this reason, the star is observed at either elongation, when it can be followed until it reaches its extreme east or west point, and correct results can be obtained independent of the time. A point is then set on the ground in the vertical plane under the star, The line from the transit to this point will not be a true meridian but must be corrected by the angle between the pole and the star at elongation, this angle being called the *azimuth of Polaris at elongation.* The observation is made as follows:

Method of Observation. *Elongation of Polaris.* By means of Table V compute the approximate time of elongation; set up the instrument and carefully level-up about 20 minutes before this time. Locate the star as described in the observation for latitude, Fig. 91, and illuminating the cross hairs as shown in Fig. 92, set the vertical wire on the star and clamp both motions, If Polaris is observed at eastern elongation, it will appear to move slowly upward and very slightly eastward until elongation is reached, when, continuing its motion upward, it will move slowly to the west. It must be followed to its most easterly position. At western elongation, its motion is downward.

Now plunge down and set a point about 300 feet away and directly under the star. It takes considerable patience to set this point correctly. A lantern should first be lined in; than a piece of white paper should be held in front of the lantern and the plumb bob in front of the paper. The illuminated paper gives a white background on which the transit cross hairs may clearly be seen, by means of which the plumb bob may be lined in and a stake and tack set.

Azimuth of Polaris at Elongation. The next morning the azimuth of Polaris at elongation is found by interpolation from Table VI, or it is computed from the formula

$$\text{sine of azimuth at elongation} = \frac{\text{sine of polar distance}}{\text{cosine of latitude}}$$ ✓

taking the polar distance of Polaris from Table IV and obtaining the latitude as already explained. If the observation is taken at eastern elongation, this angle is laid off to the west of the stake and a new point set, which gives a true meridian. If western elongation is used, the azimuth of Polaris at elongation is laid off to the east. Note the method of laying off an angle as explained under Transit, "Plane Surveying", Part II.

Determination of True Azimuth of Line. If it is not desired to establish a meridian on the ground, but simply to determine the true azimuth of a line, the observation is taken with the instrument set up at one end of the line, and the point is set vertically under Polaris at elongation as before. Later the angle clockwise between this point and the other end of the line is read—by repetition, for accurate results—and this angle is then corrected by applying the azimuth of Polaris as above.

Examples. 1. An observation was taken on Polaris on July 5, 1916. What elongation would be used, and what is the approximate time?

Ans. Use eastern elongation, which occurs at about 12:45 A.M.

2. Suppose the latitude for the above observation were 41° 40′ 00″; what is the azimuth of Polaris? Ans. 1° 31′ 51″

Observation on Sun for Meridian

Choice of Method. Two methods are available for determining a meridian by an observation on the sun, namely, by measuring the vertical angle of the sun with a transit, from which its azimuth is easily computed; and by measuring the angle by means of a solar attachment.

The first method can be used by any surveyor who occasionally has to determine a true meridian. The solar attachment method, however, is advisable only in such work as laying out public lands, where a number of determinations of meridian have to be made each day. The best time for making solar observations is from 8 to 10 A.M. and from 2 to 4 P.M.

Transit Method of Measuring Sun's Altitude. *Principle.* The transit used should preferably have a full vertical arc. The observation consists in measuring the horizontal and vertical angles of the center of the sun. The sun is observed by placing a piece of paper a few inches back of the eyepiece in the same manner as for the latitude determination given above. Instead of trying to observe the center of the sun by getting it in the position shown in Fig. 95(*a*), more accurate results can be obtained by taking two observations with the sun tangent in two diagonally opposite quadrants as shown in Fig. 95(*b*) and Fig. 95(*c*). The average of the two readings then gives the mean position of the sun.

Observation. The transit is set up at one end of the line, the azimuth of which is to be determined, and carefully leveled-up.

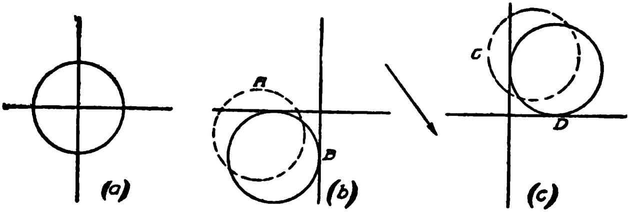


Fig. 95. Diagram Showing Method of Transit Observation on Sun for Meridian

The plates are set at zero and the instrument is sighted at the far end of the line. The upper motion is unclamped and the telescope is turned so as to sight on the sun. The direction of motion of the sun on the paper is noted. If it is moving as indicated by the arrow, Fig. 95, the two corners between the horizontal and vertical wires shown would be used. If the motion was downward to the left, the other two corners would be used. This makes it easy to get the exact setting with the horizontal and vertical wires both tangent to the sun. Thus, for Fig. 95(*b*), the horizontal wire may be set just a little below the edge of the sun, as indicated in connection with the dotted line representing the first position, and by means of the horizontal tangent, or slow-motion, screw the vertical wire may be kept tangent to the edge of the sun at *B* until the sun has moved sufficiently to be exactly tangent at A. The vertical and horizontal arcs are then read, the telescope is transited and reversed 180 degrees in azimuth—this can be done only if the instrument has a complete vertical arc, but it eliminates errors, as noted above for latitude—and the telescope is clamped so that the vertical wire is slightly ahead of the sun's image, as at C, Fig. 95(c), and is followed by the vertical-motion or horizontal-axis slow-motion screw in a manner similar to that previously described.

A complete computation for an observation of this kind is tabulated below. The solution of the formula

$$\cos A = \frac{\sin \delta}{\cos \phi \cos h} - \tan \phi \tan h$$

gives the angle from the north to the sun, measured clockwise in the morning and counterclockwise in the afternoon from which

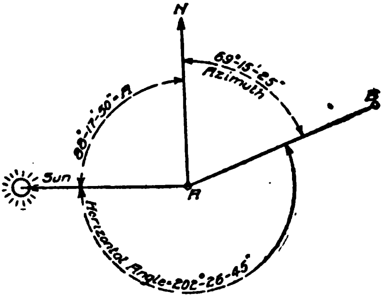


Fig. 96. Diagram for Computation of Latitude by Observation on Sun

the azimuth of the line is computed as shown in Fig. 96. Note the form of computation for the solution of this formula, which uses both natural and logarithmic functions. Also note that strict attention must be paid to the sign of the declination—which is − in winter—and to other quantities. If cosine A comes out minus, it indicates that the angle is equal to 180 degrees minus the value obtained from trigonometric tables.

Typical Observation on Sun for Meridian. *Data.* Instrument at ⊙ A; sight to ⊙ B; 4:00 P.M., June 14, 1916.

Position of Sun (See Fig. 95)	Horizontal Angle ⊙B to Sun, Clockwise	Vertical Angle of Sun
Position (*b*)	201° 57′ 00″	34° 32′ 00″
Position (*c*)	202° 56′ 30″	34° 36′ 30″
Mean position (*a*)	202° 26′ 45″	34° 34′ 15″

Vertical angle of sun, as observed	34° 34′ 15″
Correction for refraction, from Table IX	−1′ 25″
True altitude of sun = h	= 34° 32′ 50″

Declination at Greenwich noon, from *Ephemeris*	= 23° 16′ 10.5″
Hourly change in declination = +7.49″	
Time since Greenwich noon = 9 hours	
Correction to declination	= + 1′ 07.4″
Declination at 4 P.M. = δ	= 23° 17′ 18″

Latitude of place of observation = ϕ = 41° 40′ 40″ N

Computation:

Angles		Antilogs	Logs	
ϕ	41° 40′ 40″		log cos = 9.87326	log tan = 9.94952
h	34° 32′ 50″		log cos = 9.91575	log tan = 9.83790
			Sum = 9.78901	
δ	+23° 17′ 18″		log sin = 9.59700	
		+0.64268	Diff. = 9.80799	
		0.61295		
nat. cos A = Diff =		+0.02973		Sum = 9.78742

Therefore A = 88° 17′ 50″

azimuth = 360° − (88° 17′ 50″ + 202° 26′ 45″) = 69° 15′ 25″

Test of Computation. Students should study this computation in connection with the formula just given and, further, should make out a blank computation sheet in the form given above, assuming the measurements of angles, etc., and then make the entire computation without referring to the above tabulation.

Solar Attachment Method of Measuring Sun's Altitude. Several different forms of solar attachment have been devised, but the Saegmüller only will be described here as the principles involved are the same for the other forms. The attachment shown in Fig. 97 consists essentially of a small telescope mounted on a horizontal axis and revolving about a vertical or polar axis. It fits on top of the engineer's transit, as shown in Fig. 98.

The vertical axis of the solar attachment may be made perpendicular to a plane through the horizontal axis and horizontal line of sight of the transit. Also the solar telescope may be elevated to any angle with reference to the horizontal line of sight of the transit.

If the transit is turned in azimuth until the line of sight is brought into the meridian, the telescope pointing to the south, and if we lay off upon the vertical circle, upward, the colatitude of the place, Fig. 89, the vertical axis of the solar will be parallel to the axis of the earth, that is, it will be what is called a *polar axis*. If the two lines of sight are parallel and the solar telescope is revolved on its polar axis, it is evident that its line of sight will describe a plane parallel to the plane of the equator. If, now, the transit telescope still is maintained parallel to the equator and if the solar telescope is turned on its horizontal axis until the angle between the two lines of sight equals the declination of the sun, then, when the solar telescope is revolved on its polar axis, its line of sight will follow the path of the sun across the sky, provided there is no change in the sun's declination, and the solar telescope could be brought on the sun. Conversely, if the colatitude and declination are laid off as above described, and the transit and solar telescopes are revolved until the image of the sun

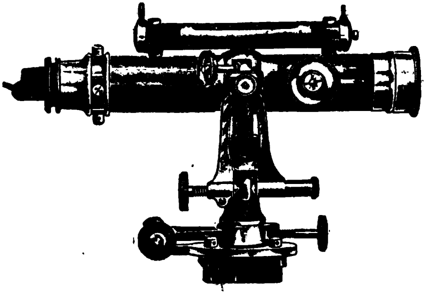


Fig. 97. Typical Solar Attachment for Transit
Courtesy of Keuffel & Esser Company, New York City

is brought into the field of the solar telescope, the line of sight of the transit telescope will be found in the meridian.

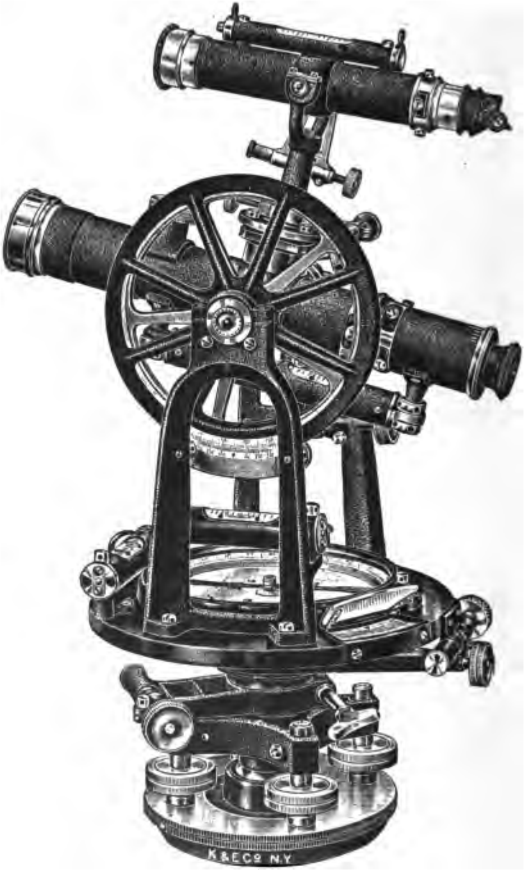


Fig. 98. Complete Engineer's Transit with Solar Attachment
Courtesy of Keuffel & Esser Company, New York City

The horizontal axis of the solar telescope and the polar axis of the solar are provided with clamps and tangent screws by means of which these angles may be carefully laid off. Two pointers are attached to the solar telescope, so adjusted that when the shadow of the one is thrown upon the other, the sun will appear in the field of view. There are also provided colored glass shades to the eyepiece to protect the eye when making observations on the sun. The objective and the cross hairs are focused in the usual way.

It is assumed in what follows that the transit is in perfect adjustment, particularly the plate levels, the horizontal axis of the telescope, the telescope bubble, and the vertical circle.

Adjustment of Polar Axis of Solar Telescope. The polar axis should be vertical when the line of sight and the horizontal axis of the telescope are horizontal. To make this adjustment, level the transit by means of the plate levels; or, if the telescope is not fitted with a level, make the vernier of the vertical circle read zero and proceed as follows: Bring the bubble of the solar to the middle of its tube and clamp the horizontal axis. Loosen the clamp of the polar axis and turn the solar on its axis through 180 degrees. If the bubble remains in the middle of the tube, the polar axis is in adjustment. If the bubble runs toward one end of the tube, correct one-half the error by the vertical-motion screw of the solar telescope and the other half by means of the capstan-headed screws at the base of the solar.

If the telescope of the transit is fitted with a level, it will be better to test the verticality of the vertical axis by means of it, since it is longer and more sensitive than the bubbles on the plate. To do this, turn the transit on its vertical axis until the long bubble of the telescope is directly over a pair of diagonally opposite plate screws and bring the bubble to the middle by means of the tangent screw attached to the horizontal axis of the telescope. Now revolve the telescope on its vertical axis through 180 degrees and note whether the bubble runs to one end; if it does, correct one-half the error by the parallel plate leveling screws and the other half by the tangent screw of the horizontal axis, and repeat this test and correction until the bubble remains in the middle, in all positions. This is known as *leveling-up with the long bubble.* The solar may then be tested as before.

Adjustment of Cross Hairs of Solar. The line of sight of the solar telescope should be parallel to the axis of the solar bubble tube. The first adjustment having been made, bring the two telescopes into the same vertical plane. Thus, sight on the corner of a building or on any well-defined point with the transit telescope and clamp the vertical axis. Now turn the telescope of the solar on the polar axis until the intersection of the cross hairs covers the same point and clamp the polar axis. Level both telescopes, by bringing the bubbles to the middle, and measure the distance between the horizontal axes of the two telescopes; draw two black parallel lines this distance apart on a piece of white paper. Tack up the paper against a wall, post, or other convenient object, adjusting it in position so that one black line is covered by the horizontal cross hair of the transit telescope. Notice whether the other black line is covered by the horizontal cross hair of the solar; if so, the adjustment is completed; otherwise, move the ring carrying the cross hairs of the solar until the second black line is covered. Adjusting the cross-hair ring may displace the solar telescope vertically, so that the bubble should again be brought to the middle of the tube and the adjustment tested and repeated until the two lines of sight are parallel when the two bubbles are simultaneously in the middle of the tubes.

Determination of True Meridian with Solar Attachment. An observation with the solar attachment involves four quantities as follows: (1) time of day, that is, the hour-angle of the sun; (2) declination of the sun; (3) latitude of the place of observation; and (4) direction of the meridian.

Any three of these quantities being known, the fourth may be determined by direct observation. The principal use of the solar attachment is to determine a true meridian when the other three quantities are known. This is done as follows:

Set-up the transit and level the instrument carefully. Bring the lines of sight of the two telescopes into the same vertical plane by the method previously described. Take the declination of the sun as given in the *Solar Ephemeris* for the given day and correct it for refraction and hourly change. Revolve the transit telescope on its horizontal axis so that the vertical circle will record this corrected declination, turning it down if the declination is north, and elevating it if the declination is south. Now, without disturbing the position of the transit telescope, bring the solar telescope to a horizontal position by means of the solar telescope level. It is evident that the angles between the lines of sight will then equal the corrected declination of the sun.

Next, without disturbing the relative positions of the two telescopes, set the vernier of the transit telescope to the colatitude of the place and clamp the horizontal axis. It is evident that the transit telescope is parallel to the equator and that the solar telescope is in a position to describe the path of the sun when the line of sight of the transit is in the true meridian; and unless the line of sight is in the true meridian, the sun cannot be brought between the cross hairs of the solar telescope. Therefore, unclamp the vertical axis of the transit and the polar axis of the solar and, maintaining the relative positions of the telescopes, revolve the transit on its vertical axis and the solar on its polar axis until the sun is brought between the cross hairs of the solar telescope. Now clamp the vertical axis of the transit which is pointing in the true meridian.

Preparation of Declination Settings for Day's Work. The *Solar Ephemeris* gives the declination of the sun for the given day for Greenwich mean noon. Since all points in America are west of Greenwich by 4, 5, 6, 7, or 8 hours, the declination found in the *Ephemeris* is the declination at the given place at 8, 7, 6, 5, or 4 A.M. of the same date, according to whether the place lies in the Colonial, eastern, central, mountain, or Pacific time belts, respectively.

The refraction corrections will be different than for the direct method as the correction required is for the declination, not the altitude. It is given in Table IX for intervals of 4 or 5 days for latitude 40°. If the latitude is more or less than 40°, these corrections are to be multiplied by the corresponding latitude coefficient, Table X. These corrections are also given in many *Solar Ephemerises*. (See Table VIII.)

Thus the refraction corrections in latitude 30° are 0.65 and those of 50° are 1.42 of the corresponding ones in latitude 40°. There is an error in this method which is small except when the sun is very near the horizon, and then any refraction becomes very uncertain.

Declination is north (positive) from March 21 to September 23, inclusive, and is south (negative) from September 24 to March 20,

TABLE IX

Refraction Corrections for Declination

(Latitude, 40°)

JANUARY		
da.	hr.	Corr.
		′ ″
1	1	1 58
2	2	2 16
3	3	3 04
4	4	6 23
5		
	1	1 54
6	2	2 11
7		
8	3	2 59
9	4	6 01
10		
	1	1 50
11	2	2 06
12		
13	3	2 49
14	4	5 33
15		
	1	1 46
16	2	2 01
17		
18	3	2 40
19	4	5 00
20		
	1	1 42
21	2	1 56
22		
23	3	2 31
24	4	4 35
25		
	1	1 37
26	2	1 50
27		
28	3	2 22
29	4	4 07
30		
	1	1 32
31	2	1 44

FEBRUARY		
da.	hr.	Corr.
		′ ″
1	3	2 13
2		
3	4	3 41
4		
5	1	1 26
	2	1 37
6		
7	3	2 04
8	4	3 21
9		
10	1	1 21
	2	1 31
11		
12	3	1 56
13	4	3 04
14		
15	1	1 16
	2	1 25
16	3	1 48
17	4	2 47
18	5	8 39
19		
20	1	1 12
	2	1 20
21	3	1 40
22	4	2 31
23	5	6 49
24		
25	1	1 07
	2	1 15
26	3	1 33
27	4	2 18
28	5	5 29

MARCH		
da.	hr.	Corr.
		′ ″
1	1	1 03
2	2	1 10
3	3	1 27
4	4	2 06
5		
	1	0 59
6	2	1 06
7	3	1 21
8	4	1 56
9	5	4 04
10		
	1	0 55
11	2	1 02
12	3	1 15
13	4	1 47
14	5	3 34
15		
	1	0 52
16	2	0 58
17	3	1 10
18	4	1 39
19	5	3 08
20		
	1	0 48
21	2	0 54
22	3	1 05
23	4	1 32
24	5	2 51
25		
	1	0 45
26	2	0 50
27	3	1 01
28	4	1 25
29	5	2 34
30		
	1	0 42
31	2	0 47

APRIL		
da.	hr.	Corr.
		′ ″
1	3	0 57
2	4	1 19
3	5	2 18
4		
5	1	0 39
	2	0 44
6	3	0 54
7	4	1 14
8	5	2 08
9		
10	1	0 36
	2	0 41
11	3	0 51
12	4	1 10
13	5	1 58
14		
15	1	0 34
	2	0 38
16	3	0 48
17	4	1 06
18	5	1 49
19		
20	1	0 32
	2	0 36
21	3	0 45
22	4	1 02
23	5	1 42
24		
25	1	0 30
	2	0 34
26	3	0 42
27	4	0 58
28	5	1 36
29		
30	1	0 28
	2	0 32

MAY		
da.	hr.	Corr.
		′ ″
1	3	0 39
2	4	0 55
3	5	1 30
4		
5	1	0 26
	2	0 30
6	3	0 37
7	4	0 53
8	5	1 26
9		
10	1	0 25
	2	0 29
11	3	0 36
12	4	0 51
13	5	1 22
14		
15	1	0 23
	2	0 27
16	3	0 34
17	4	0 49
18	5	1 18
19		
20	1	0 22
	2	0 26
21	3	0 33
22	4	0 47
23	5	1 15
24		
25	1	0 21
	2	0 25
26	3	0 32
27	4	0 46
28	5	1 13
29		
30	1	0 20
	2	0 24
31	3	0 31

JUNE		
da.	hr.	Corr.
		′ ″
1	4	0 44
2	5	1 11
3		
4	1	0 19
5	2	0 23
	3	0 30
6	4	0 43
7	5	1 10
8		
9	1	0 18
10	2	0 22
	3	0 29
11	4	0 43
12	5	1 09
13		
14	1	0 18
15	2	0 22
	3	0 29
16	4	0 42
17	5	1 08
18		
19	1	0 18
20	2	0 22
	3	0 29
21	4	0 42
22	5	1 08
23		
24	1	0 18
25	2	0 22
	3	0 29
26	4	0 42
27	5	1 08
28		
29	1	0 18
30	2	0 22
	3	0 29

JULY		
da.	hr.	Corr.
		′ ″
1	4	0 43
2	5	1 09
3		
4	1	0 19
5	2	0 23
	3	0 30
6	4	0 43
7	5	1 10
8		
9	1	0 20
10	2	0 24
	3	0 31
11	4	0 44
12	5	1 11
13		
14	1	0 21
15	2	0 25
	3	0 32
16	4	0 46
17	5	1 13
18		
19	1	0 22
20	2	0 26
	3	0 33
21	4	0 47
22	5	1 15
23		
24	1	0 23
25	2	0 27
	3	0 34
26	4	0 49
27	5	1 18
28		
29	1	0 25
30	2	0 29
	3	0 36
31	4	0 51

AUGUST		
da.	hr.	Corr.
		′ ″
1	5	1 22
2		
3	1	0 26
4	2	0 30
5	3	0 37
	4	0 53
6	5	1 26
7		
8	1	0 28
9	2	0 32
10	3	0 39
	4	0 55
11	5	1 30
12		
13	1	0 30
14	2	0 34
15	3	0 42
	4	0 58
16	5	1 36
17		
18	1	0 32
19	2	0 36
20	3	0 45
	4	1 02
21	5	1 42
22		
23	1	0 34
24	2	0 38
25	3	0 48
	4	1 06
26	5	1 49
27		
28	1	0 36
29	2	0 41
30	3	0 51
	4	1 10
31	5	1 58

SEPTEMBER		
da.	hr.	Corr.
		′ ″
1	1	0 30
2	2	0 44
3	3	0 54
4	4	1 14
5	5	2 08
6	1	0 42
7	2	0 47
8	3	0 57
9	4	1 19
10	5	2 18
11	1	0 45
12	2	0 50
13	3	1 01
14	4	1 25
15	5	2 34
16	1	0 48
17	2	0 54
18	3	1 05
19	4	1 32
20	5	2 51
21	1	0 52
22	2	0 58
23	3	1 10
24	4	1 39
25	5	3 08
26	1	0 55
27	2	1 02
28	3	1 15
29	4	1 47
30	5	3 34

OCTOBER		
da.	hr.	Corr.
		′ ″
1	1	0 59
2	2	1 06
3	3	1 21
4	4	1 56
5	5	4 04
6	1	1 03
7	2	1 10
8	3	1 27
9	4	2 06
10	5	4 39
11	1	1 07
12	2	1 15
13	3	1 33
14	4	2 18
15	5	5 29
16	1	1 12
17	2	1 20
18	3	1 40
19	4	2 31
20	5	6 49
21	1	1 16
22	2	1 25
23	3	1 48
24	4	2 47
25	5	8 39
26	1	1 21
27	2	1 31
28	3	1 56
29	4	3 04
30	5	11 01
31		

NOVEMBER		
da.	hr.	Corr.
		′ ″
1	1	1 26
2	2	1 37
3		
4	3	2 04
5	4	3 21
6	1	1 32
7	2	1 44
8		
9	3	2 13
10	4	3 41
11	1	1 37
12	2	1 50
13		
14	3	2 22
15	4	4 07
16	1	1 42
17	2	1 56
18		
19	3	2 31
20	4	4 35
21	1	1 46
22	2	2 01
23		
24	3	2 40
25	4	4 59
26	1	1 50
27	2	2 06
28		
29	3	2 49
30	4	5 33

DECEMBER		
da.	hr.	Corr.
		′ ″
1	1	1 54
2	2	2 11
3		
4	3	2 59
5	4	6 01
6	1	1 58
7	2	2 26
8		
9	3	3 04
10	4	6 23
11	1	2 00
12	2	2 19
13		
14	3	3 09
15	4	6 38
16	1	2 01
17	2	2 20
18		
19	3	3 11
20	4	6 47
21	1	2 01
22	2	2 20
23		
24	3	3 11
25	4	6 49
26	1	2 00
27	2	2 19
28		
29	3	3 09
30	4	6 43
31	(See Jan. 1)	

TABLE X

Latitude Coefficients

Latitude (°)	Coefficient	Latitude (°)	Coefficient	Latitude (°)	Coefficient	Latitude (°)	Coefficient
15	0.30	27	0.56	39	0.96	51	1.47
16	0.32	28	0.59	40	1.00	52	1.53
17	0.34	29	0.62	41	1.04	53	1.58
18	0.36	30	0.65	42	1.08	54	1.64
19	0.38	31	0.68	43	1.12	55	1.70
20	0.40	32	0.71	44	1.16	56	1.76
21	0.42	33	0.75	45	1.20	57	1.82
22	0.44	34	0.78	46	1.24	58	1.88
23	0.46	35	0.82	47	1.29	59	1.94
24	0.48	36	0.85	48	1.33	60	2.00
25	0.50	37	0.89	49	1.38		
26	0.53	38	0.92	50	1.42		

inclusive. The hourly changes are positive from December 22 to June 21, inclusive, and negative from June 22 to December 21, inclusive. The refraction correction is always positive, that is, it always increases numerically the north declination and diminishes numerically the south declination.

The solar apparatus should not be used between 11 A.M. and 1 P.M. if the best results are desired. From 7 to 10 P.M. and from 2 to 5 P.M. in the summer will give the best results. The greater the hour-angle of the sun the better the observation will be, so far as instrumental errors are concerned, but the position of the sun must not be observed when it is too close to the horizon, as the uncertainties due to refraction will cause unknown errors of considerable magnitude.

ILLUSTRATIVE EXAMPLE

Let it be required to prepare a table of declination for June 10, 1916, for a point whose latitude is 40° 20′, and which lies in the central time belt.

The *Solar Ephemeris* is based upon Greenwich mean noon and latitude 40° north.

For use at any other place the declination of the sun at noon on the stated day must be reduced to the time and place of the desired reading (corrected for refraction).

The refraction corrections are given for certain hour-angles 1, 2, 3, 4, and 5, which refer to the hours before and after Greenwich, noon, respectively. For instance the refraction correction for the 2-hour period refers to either 10 A.M. or 2 P.M., Greenwich time; the 3-hour period for either 9 A.M. or 3 P.M., etc. The refraction correction values must further be corrected for latitude in order to reduce them to the altitude of the place at which the readings are to be used. This is done by use of the latitude coefficients, Table X.

The example calls for June 10, 1916. Consulting the *Ephemeris* table we find the declination as N 23° 00′ 54″. The time belt is central which, by reference to the "time belt chart", indicates an average longitude of 90° west, which is 6 hours earlier than Greenwich time since each 15° of longitude is equivalent to one hour of time. The place being westerly from Greenwich is of course earlier Then applying this correction we have mean noon at Greenwich as 6 A.M. at our longitude. As it is summertime the sun is above the horizon and the reading may possibly be used. You will note the tables give corrections for refraction for from 1 to 5 hours time on each side of noon, the 1-hour correction being applicable to either 11 A.M. or 1 P.M., 2-hour for either 10 A.M. or 2 P.M., etc. In winter the 4-hour limit is given as you cannot see well enough before 8 P.M. nor after 4 P.M., as a rule.

Now proceeding to compile our readings, we have for 6 A.M. a declination of N 23° 00′ 54″ and an hourly correction of 11.6″, which is positive, as the declination increases from zero north at the spring equinox, March 20, until it is a maximum, or 23° 27′ at the summer soltice, June 21; after this it decreases until the fall equinox, September 22, when it is again zero and passes to south declination. It keeps increasing southerly until the winter soltice, December 22, when it is at its maximum south declination from which time it decreases until it reaches the equinox again on March 20. From this you will note the declination is

Increasing Mar. 21 to June 21
Increasing June 22 to Sept. 22
Increasing Sept. 23 to Dec. 22
Increasing Dec. 23 to Mar. 20

and is

N (+) Mar. 21 to Sept. 22
S (−) Sept. 23 to Mar. 20

The refraction correction is always plus. Taking, now, our declination at 6 A.M. as 23° 00′ 54″ and adding our hourly correction of 11.6″, gives the sun's apparent declination (neglecting refraction) for 7 A.M. as 23° 01′ 06″; for 8 A.M., 11.6″ more than 7 A.M., or 23° 01′ 17″; for 9 A.M., 23° 01′ 29″; for 10 A.M., 23° 01′ 40″; for 11 A.M., 23° 01′ 52″; for noon, 23° 02′ 04″, etc. As a check on your detail work, multiply the hourly change by the number of hours and add to the starting declination; that is, as we have used 6 hours, our total change from 23° 00′ 54″ will be 6×11.6″=69.6″, say, +1′ 10″, which added to the above is 23° 02′ 04″, which verifies our work.

Next in order will be the refraction correction. As the latitude is 40° 20′ north the correction coefficient must be interpolated, the tabular values (Table X) being for each degree, whence if *C* equals correction coefficient desired, we have

$$C=1.00+\frac{0.04}{3}=1.013$$

The refraction correction for the 1st, 2d, 3rd, 4th, and 5th hours on either side of noon will be the tabular values 18″, 22″, 29″, 43″, and 69″ multiplied respectively by the coefficient 1.013, or

18″×1.013=18.234″, say, +18″ for 1 A.M. or 1 P.M.
22″×1.013=22.286″, say, +22″ for 10 A.M. or 2 P.M.
29″×1.013=29.377″, say, +29″ for 9 A.M. or 3 P.M.
43″×1.013=43.559″, say, +44″ for 8 A.M. or 4 P.M.
69″×1.013=69.897″, say, +70″ for 7 A.M. or 5 P.M.

Summing up, we have

Hour	Declination	Refraction Correction	Setting
7 A.M.	N 23° 01′ 06″	+1′ 10″	N 23° 02′ 16″
8 A.M.	23° 01′ 17″	0′ 44″	23° 02′ 01″
9 A.M.	23° 01′ 29″	0′ 29″	23° 01′ 58″
10 A.M.	23° 01′ 40″	0′ 22″	23° 02′ 02″
11 A.M.	23° 01′ 52″	0′ 18″	23° 02′ 10″
12 Noon	23° 02′ 04″	....	23° 02′ 04″
1 P.M.	23° 02′ 15″	0′ 18″	23° 02′ 33″
2 P.M.	23° 02′ 27″	0′ 22″	23° 02′ 49″
3 P.M.	23° 02′ 38″	0′ 29″	23° 03′ 07″
4 P.M.	23° 02′ 50″	0′ 44″	23° 03′ 34″
5 P.M.	23° 03′ 02″	1′ 10″	23° 04′ 12″

LAND SURVEYING

Object of Land Surveying. Land surveying has as its object either the obtaining of necessary data for writing a description of a piece of land—such as is used in a deed when the ownership of the land changes—and finding its area or the reverse operation of finding and marking the boundaries of a described tract. In connection with most land surveys there is almost always a plat made showing the results of the survey. Land surveys may be divided into farm-land, city, and public-land surveys.

FARM SURVEYS

Essentials of Survey. In the eastern part of the United States farm land is sold in more or less irregular tracts usually described by "metes and bounds." It very often happens that these lands have never been surveyed and have been described for many years by reference to certain landmarks that in many cases have disappeared. When called upon to make a survey in such a case, the surveyor should first set permanent marks or monuments at the corners of the property as shown him and agreed upon by the adjoining property owners. These monuments are too frequently of a temporary nature, soon lost or destroyed, and the surveyor should insist upon setting truly permanent marks of stone, concrete, or iron. These marks should also be *witnessed* by noting the bearing and distance from the mark to trees and other natural objects that will aid in finding the mark or in locating the corner in case the mark is removed or destroyed.

The surveyor then proceeds to make his survey and finally to obtain the lengths and true bearings of all boundaries, and also the area. A *plat* of these results should show the following:

(1) The lengths of all lines.
(2) The true bearings of all lines.
(3) The exact position and character of all monuments set, with notes of reference points.
(4) The names of all streets, streams, or landmarks.
(5) The direction of the true meridian, and a note that true bearings are shown.
(6) The scale, a simple and explicit title, the date, and the signature of the surveyor.

In addition, it is well for the surveyor to furnish a certificate stating how the survey has been made, that the map is a correct representation of the tract, and that the monuments have been set. Also an agreement should be drawn up and signed by all adjoining property owners agreeing to the boundaries shown on the map. If these simple rules had always been and were always followed, a large amount of the trouble and litigation in regard to ownership of land would be done away with.

Instruments and Accuracy. The instruments used at present are usually the transit and tape. As already noted in "Plane Surveying", Part II, the compass and chain were formerly used, and the compass is still the best instrument in making surveys of wooded tracts where the land is not very valuable. The accuracy the surveyor should obtain must, for reasons of economy, be made to vary with the value of the land to be surveyed. For work in large cities where a square foot of land is frequently worth more than an acre of land in the country, it is obvious that the survey should be made with great care. For land worth $20 or thereabout per acre and wooded, the compass could be used, and an accuracy of 1 in 700 to 1000 should be obtained. When the transit is used, an accuracy of 1 in 2000 is easily obtained and should suffice for ordinary farm surveys. In suburban districts 1 in 5000 or even 1 in 10,000 can be obtained with the transit and tape, using the azimuth method, while for city work, 1 in 30,000 or even 1 in 50,000 is desirable, and this requires reading the angles by repetition and making allowances for temperature, sag, and pull in the taping.

Direct Survey with Transit at Corners. When the farm corners can be occupied by the transit and the boundaries can be measured directly, the work of making the survey is very simple. A true meridian is determined with the instrument at one corner of the property, and a closed azimuth traverse is run around the boundaries, starting from this point, all lengths being measured with the tape. The error of closure of this traverse is then computed, and if it comes within the limit allowed ("Plane Surveying", Part II, page 76), the latitude and longitude differences are balanced and the area computed as in the example already given. Finally the lengths and bearings of the sides are computed from the balanced latitude and longitude differences and are recorded on the plat. The boundaries thus obtained form the sides of a figure that closes exactly, and the boundaries shown on a land survey map should always close in this way.

Indirect Survey. *Method of Offsets.* When the survey cannot be run over the actual boundaries, owing to fences or other obstructions, two methods are available. The first method is by offsets and, while it saves quite a large amount of computation, it requires considerable more time in the field than the auxiliary polygon method. In making a survey by offsets, each corner is offset a certain amount at right angles to the sides, and the offset side is then measured. Thus, in Fig. 99, suppose the actual corners of the property are *1*, *2*, *3*, and *4*, and that all sides but *2–3* are obstructed by fences so it is impossible to set-up the instrument at *1* or *4* or to measure the lengths *3–4*, *4–1*, or *1–2* directly. Equal offsets, *1–B* and *2–C*, are then made at *1* and *2* and the length *B–C*, which evidently equals *1–2*, is measured. In a similar way the lengths of *3–4* and *4–1* are found. If a compass is used, it can be set at *B* and *C* and a back and front bearing of *B–C* obtained which will also equal that of *1–2*, as *B–C* is parallel to *1–2*. If a transit is used, following the azimuth method, it will be necessary to set the transit up at the points of intersection of the offset sides in order that a closed traverse may be formed, as the azimuth must be carried on from one set-up of the instrument

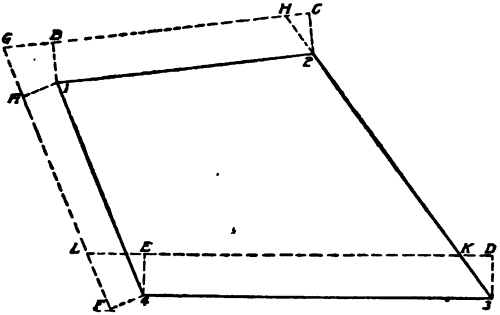


Fig. 99. Diagram of Survey by Offsets

to the next. Thus, in Fig. 99, the transit would be set-up, say, at *G*, from which point a meridian has been determined and would be moved to *H*, *K*, and *L*, closing back to *G*. This involves considerable field work in offsetting and prolonging lines and is seldom an economical method.

Method of Auxiliary Traverse. Taking the same field as above, this method is illustrated by Fig. 100. Starting at *2* a closed azimuth traverse is run from *2–3*, *3–B*, *B–C*, *C–D*, *D–A*, closing with *A–2*. The points *A*, *B*, *C*, and *D* are any convenient points. *A* and *C* should generally be within 100 feet (a tape length) of *1* and *4* as tie lines. *A–1* and *4–C* are measured and bearings read from these points; there is no check on their accuracy, hence they should be short and carefully measured. These points should also be picked out so as to make the chaining easy and to avoid the necessity of clearing the lines.

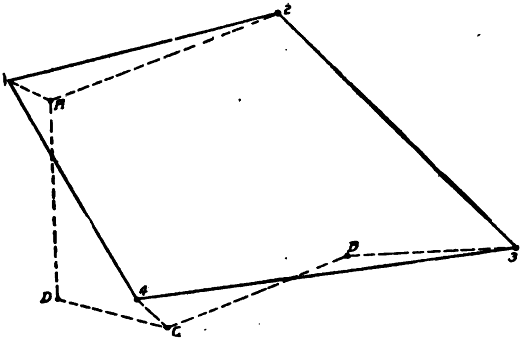


Fig. 100. Diagram of Survey by Auxiliary Traverse

The traverse *2–3–B–C–D–A–2* is now computed and the error of closure found. If this is within satisfactory limits, the latitude and longitude differences are then balanced. Now, in order to find the length and bearing of *1–2*, for example, the missing-course method should be used. Thus *1–2* is the closing side of a traverse *2–A–1–2*, in which the lengths and bearings of all the other sides, namely, the tie line *A–1* and the auxiliary traverse side *A–2*, are known. Hence, using the balanced latitude and longitude differences for *A–2*, the bearing and length of *1–2* can be easily computed by the method of latitude and longitude differences already described. The lengths and bearings so computed will form the sides of a figure that will close exactly as required.

Map of Survey. In Fig. 101 is shown the general form in which the map would be drawn up. In plotting surveys the bearings are always given in the direction in which they are written.

Description of Survey. Supposing the land, Fig. 101, belongs to John Brown, a description would be written as follows:

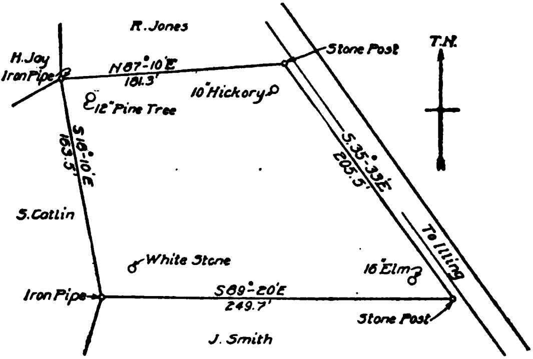


Fig. 101. Typical Map of Farm Survey

Beginning at a stone post on the west side of the road from Brownsville to Illing at the N E corner of the land of J. Smith and about 300 feet north of the railroad crossing of said road and the Western Railroad, from which post an elm tree, 16 inches in diameter, bears N 62° W, 20 feet, and running thence N 89° 20′ W, along the north line of said J. Smith, two hundred forty-nine and seven-tenths (249.7) feet to an iron pipe, from which a large white stone bears N 20° E, 12 feet; thence N 18° 10′ W, along the east line of S. Catlin, one hundred sixty-three and five-tenths (163.5) feet to an iron pipe, from which a pine tree, 12 inches in diameter, bears S 47° E, 21 feet; thence N 87° 10′ E, along the south line of R. Jones, one hundred eighty-one and three-tenths (181.3) feet to a stone post on the west side of said road from Brownsville to Illing, from which a hickory tree, 10 inches in diameter, bears S 18° W, 17 feet; thence S 35° 33′ E, along the west side of said road, two hundred five and five-tenths (205.5) feet to the place or point of beginning, containing one and seventeen one-hundredths (1.17) acres, more or less, the bearings in this description being all true bearings.

Example. Write out a description similar to the foregoing, using the data shown in Fig. 60, "Plane Surveying", Part II, and assuming names, etc.

RESURVEYS

Difficulties. Resurveys are far more difficult than original surveys and consist in retracing an original survey on the ground from a description similar to but seldom as complete as that given above. In many cases the monuments have been destroyed, the original survey is in error, the bearings given are magnetic and hence have changed, because of change in declination, and conflicting testimony is given by interested owners as to where the original corners were. The surveyor should remember that nothing absolutely insures the correctness of his work but the finding of the old corners. These points must be accepted whether or not the bearings and distances given in the deed are correct.

The old description is probably in chains and links, and it is best to first change these to feet, and then to plot the survey to see if the description gives a closed traverse and to become familiar with the boundaries. A careful study is then made of the tract to locate, if possible, the corners described. It would be impossible to discuss all the difficulties that may occur, but a few illustrations of the methods used are as follows:

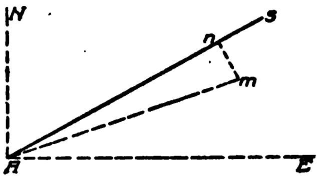


Fig. 102. Diagram Showing Method of Locating Lost Corner of Survey

Two Adjacent Corners Found. If two adjacent corners, which establish one of the old lines, can be found, the first step would be to determine its bearing and in this way find the change in declination and check the value computed from tables or a diagram. Suppose the result of the computation leads to N 38° 15′ E as the bearing. Starting at corner *A*, Fig. 102, the surveyor runs a random line AS on the bearing N 38° 15′ E and measures along this line the given distance of 2112 feet to point *S*. On arriving at *S*, the surveyor proceeds to look over the ground on both sides of this point for a lost corner, which is described in the old record as a monument, or some other well-defined mark. If, after diligent search, no trace of this mark can be found, nothing further can be done from the data at hand. However, should the mark be found at *m*, a perpendicular is dropped on the line AS, and its length is measured, as is also the distance nS. It is now evident that the distance An becomes known. From the right triangle, the angle nAm can be computed and the present magnetic bearing of Am can be determined.

For example, suppose that mn is found to be 37.4 feet, while An is 2110.5 feet, then

$$\tan nAm = \frac{mn}{An} = 0.01772$$

in which nAm is 1° 01′, and the present magnetic bearing of Am is N 39° 16′ E. The distance $Am = \frac{2110.5}{\cos 1^\circ\ 01'}$, or 2110.84 feet. This indicates that the old survey was in error by 1.16 feet, but by reason of laws governing resurveys, the monuments or corners of the survey are the true corners, no matter how incorrectly located; therefore, the new measurement of the line should replace the old one known to be incorrect, that is, the new record of line Am is N 39° 16′ E, 2110.84 feet. Intermediate points of the line Am may now be established from the starting point A, running it out with the new bearing.

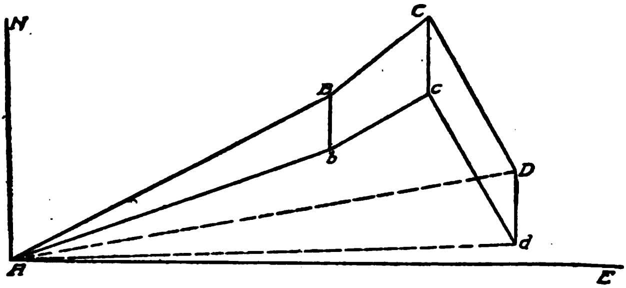


Fig. 103. Diagram Showing Method of Locating Missing Corners

To Locate Missing Corners. It sometimes happens that adjacent corners can be found while others cannot. In such cases a series of random lines may be run with the old bearings or with the old bearings corrected for the change in declination of the needle between the two dates.

As an example, Fig. 103, let the records in an old deed give the length and bearings of three lines as follows:

Ab	N 60° E	10 chains
bc	N 45° E	4 chains
cd	S 45° E	8 chains

There being no definite data at hand to determine the change in the magnetic declination between the dates of the two surveys, the lines AB, BC, and CD are run with the given bearings and distances from the known corner A. The old corners b and c cannot be found; but on arriving at D, the old corner d is discovered at a point 20.4 links south and 12 degrees west from D. It is required to locate the old corners b and c.

By the method of latitude and longitude differences, as previously explained, the lengths of the lines DA and dA may be computed. They are: for DA, S 82° 47′ W, 17.29 chains; for dA, S 83° 26′ W, 17.22 chains.

Now the error Dd between the two corners is due to two causes, viz, the continued variation in the magnetic bearings of the old surveys; and the difference in the length of the chains used. The first cause alters the polygon $AbcdA$ around the point A by a small angle. The second cause alters the length of the sides in a constant ratio. The difference between the bearings DA and dA is the constant angle, while the ratio of the length of the old lines is the constant ratio. To find the bearings of the old line, therefore, each of the given bearings should be corrected by the amount $83° 26′ - 82° 47′$, or 0° 39′. To find the length of the old line, multiply each of the given lengths by $\frac{17.22}{17.29}$, or 0.996.

Suppose now that the work of computation has been done with such precision that the error in chaining must be regarded as lying in the old survey. Applying these results, we find the adjusted bearings and lengths of the old line to be

Ab	N 60° 39′ E	9.96 chains
bc	N 45° 39′ E	3.99 chains
cd	S 44° 21′ E	7.97 chains

With the new data, the line may be rerun and the corners b and c

located, a check on the field work being that the lost line should end exactly at d.

It is, however, not difficult to compute the length and bearings of Bb and Cc, so that b and c may be located from the points B and C.

Since the angle BAb is small, the triangle BAb may be considered similar to the triangle DAd. We will then have the proportion

$$Bb : Dd :: AB : AD$$

or

$$Bb = Dd \times \frac{AB}{AD} = \frac{20.4 \times 10}{17.29} = 11.8 \text{ links}$$

A similar proportion may be written for the side Cc, and the result added to the value of Bb.

The same principle may be used to determine the bearings of Bb and Cc, so that, with the lengths and bearings of these lines determined, the probable location of the old corners b and c can be fixed.

Example. The records of an old survey read as follows: "Commencing at a point marked *No. 5* and running N 62° E 14 chains to a stake marked *A*, thence running N 43½° E 8 chains to a stake marked *B*, thence N 5° W 12 chains to a stake *C*, thence N 72½° E 10.25 chains to a stake *D*, thence S 12° W 6.43 chains to a stake marked *No. 3*. On running the lines, the end of the last one, instead of being at a stone marked *No. 3*, was 0.62 chain due E from it." Find the adjusted bearings and lengths of the old lines; also find the distance and direction from each station of the new survey to the corresponding corner of the old.

DIVISION OF LAND.

Dividing Property into Parts Having Certain Area. Another problem in land surveying that sometimes arises is the division of a piece of property into parts, each part to have a certain area. The problem is usually as follows: Given the lengths and bearings of the sides of a field containing a certain area; it is required to divide the area into certain parts by a line running in a certain direction, in which case it is necessary to determine the starting point of the dividing line. Or, it is required that the line shall begin at a certain point, in which case it is necessary to determine the direction of the line.

ILLUSTRATIVE EXAMPLES

1. A certain field is described as follows:

1–2	N 63° 51′ W	6.91 chains
2–3	N 63° 44′ W	7.26 chains
3–4	N 69° 35′ W	3.34 chains
4–5	N 77° 50′ W	6.54 chains
5–6	N 31° 24′ E	14.38 chains
6–7	N 31° 18′ E	16.81 chains
7–8	S 68° 55′ E	13.64 chains
8–9	S 68° 42′ E	11.54 chains
9–1	S 33° 45′ W	31.55 chains

Beginning at a point on the side *9–1*, it is required to divide the area into two equal parts of 37 acres by a line having a bearing of N 68° 46′ W.

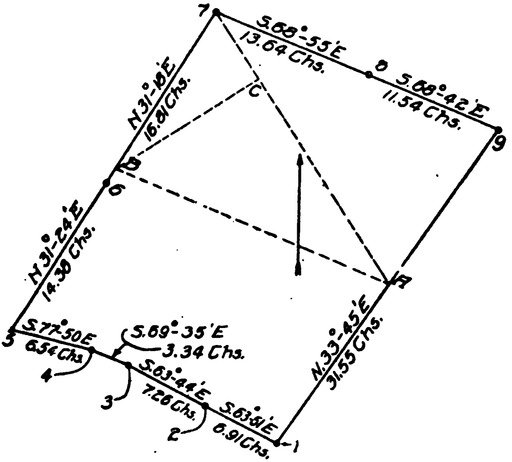


Fig. 104. Map of Survey Showing Method of Dividing Property

The plat is first drawn in the form of a map as shown in Fig. 104, and, after a few rough calculations, a trial dividing line *AB* is drawn. The distance *A–9* is then scaled off and found to be 15 chains. By the missing-side method, the lengths of *A–B* and *B–7* are then computed, giving 24.525 and 14.850 chains, respectively. This can best be done by first solving the traverse *7-8-9-A-7* for the length and bearing of *A-7* by latitude and longitude differences, and then solving the triangle *A-B-7* for the required lengths. The computed values are checked by noting that the latitude and longitude differences N and S and E and W should check, as shown in the computation tabulated below.

The area of the tract *7-8-9-A-B-7* is then computed by double longitudes, resulting in an area of 36.3919 acres, as follows:

Course	Bearing	Distance (ch.)	Latitude Difference		Longitude Difference		Double Longitude	Double Areas	
			N +	S −	E +	W −		+Products	−Products
9-A	S 33° 45′ W	15.00	......	12.472	......	8.334	− 8.334	103.942	....
A-B	N 68° 46′ W	24.525	8.882	......	......	22.860	−39.528	....	351.088
B-7	N 31° 18′ E	14.850	12.689	......	7.715	......	−54.673	....	693.736
7-8	S 68° 55′ E	13.64	......	4.907	12.727	......	−34.231	167.971	....
8-9	S 68° 42′ E	11.54	......	4.192	10.752	......	−10.752	45.072	....
Total	....	....	21.571	21.571	31.194	31.194	....	316.985	1044.824

−1044.824
+316.985
2|727.839
363.919
sq. ch., or
36.3919
acres

This is short of the desired acreage by 0.6081 acre. Our line *AB* must therefore be moved parallel to itself such a distance that the area included between its present and its new position shall equal 6.081 square chains. This is shown in Fig. 105, in which *AB* is the trial line, *A′B′* the final line, and the area *A* of *A-B-C′-A′* is to be 6.081 square chains. The perpendicular distance x may be found from the formula

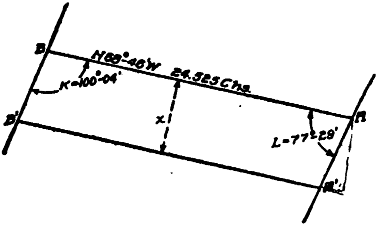


Fig. 105. Diagram Showing Adjustment of Line *AB* for Survey, Fig. 104

$$x = AB \times y + \sqrt{AB^2 y^2 - 2Ay}$$

in which – $y = 1 + (\cot K + \cot L)$

In the present case x is found to be 0.247 chain, and $BB' =$ 0.247 ÷ cos 10° 04′, or 0.251 chain; while $AA' = 0.247 \div \cos 12° 31′$, or 0.253 chain; also $A'B' = 24.525 - 0.247 \cdot \tan 12° \; 31' + 0.247 \tan 10° \; 04'$, or 24.514 chains. As a check, the area should again be computed, as shown in the following tabulation. The line *9–A* will be 15.00+0.253, or 15.253; and *B–7*=14.850+0.251, or 15.101; while *7–8* and *8–9* will remain unchanged. Here the latitude and longitude differences again afford a check, and the new area is found to be within 0.003 acre of the required value. As *A'B'* is 24.536 chains long, this would require a further movement of 0.03÷24.536, or 0.001 chain, or *x* should have been 0.248 chain.

Course	Bearing	Distance (ch.)	Latitude Difference		Longitude Difference		Double Longitude	Double Areas	
			N+	S−	E+	W−		+Products	−Products
9–A	S 33°45′W	15.253	.....	12.682	.....	8.474	− 8.474	107.467	......
A–B	N 68°46′W	24.514	8.878	.....	.....	22.850	−39.798	......	353.327
B–7	N 31°18′E	15.101	12.903	.....	7.845	.....	−54.803	......	707.123
7–8	S 68°55′E	13.64	.....	4.907	12.727	.....	−34.231	167.971	......
8–9	S 68°42′E	11.54	.....	4.192	10.752	.....	−10.752	45.072	......
Total	....	.....	21.781	21.781	31.324	31.324	....	320.510	1060.450

−1060.450
+320.510
2 |739.940
369.970
sq. ch. or
36.997
acres

2. Fig. 104 may also be used to illustrate the second type of problem. Thus, referring to Fig. 104, starting at a point *A* on *9–1*, 12.25 chains from station *9*, it is required to find the length and bearing of *AB* such that the area *9–A–B–7–8–9* shall equal 32 acres.

First draw a line from *A* to station *7*, and by latitudes and departures calculate the area *A–7–8–9–A*, and determine the length and bearing of *A–7*. Call this latter area *H*. Then the area *A–B–7–A* must equal (*32–H*.) From the point *B*, erect a perpendicular *B–C* to the line *A–7*. Then the area

$$A\text{–}B\text{–}7\text{–}A = \frac{A\text{–}7}{2} \times BC = (32 - H)$$

Therefore

$$BC = \frac{2(32 - H)}{A\text{–}7}$$

Also, since *B–C–7* is a right-angle triangle by construction

$$B\text{–}7 = \frac{BC}{\sin \, B\text{–}7\text{–}C}$$

In the triangle *A–B–7–A* we now have two sides and the included angle from which to calculate the length and bearing of *AB*.

Most of the other problems that arise are simple matters of trigonometry and can easily be worked out by the surveyor.

PUBLIC-LAND SURVEYS

Early Methods of Making Surveys. The first surveys of the public lands of the United States were carried out in Ohio, under an act of Congress approved May 20, 1785. This act provided for townships 6 miles square, containing 36 sections, 1 mile square. The idea was to do away with the old system of irregular boundaries, to simplify the description of property and to avoid litigation and boundary disputes. The townships, 6 miles square, were laid out in ranges extending northward from the Ohio River, the townships being numbered from south to north and the ranges from east to west. The territory embraced in these early surveys forms a part of the present state of Ohio and is known as "The Seven Ranges". The sections were numbered from 1 to 36, as shown in Fig. 106(*a*).

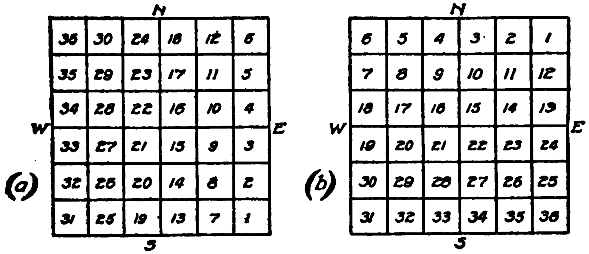


Fig. 106. Diagram Showing Method of Numbering Sections in Old Land Surveys

A subsequent act of Congress, approved May 18, 1796, provided for the appointment of a surveyor general and directed the survey of the lands northwest of the Ohio River and above the mouth of the Kentucky River. This act provided that "the sections shall be numbered, respectively, beginning with the number one in the northeast section and proceeding west and east alternately through the township with progressive numbers till the thirty-sixth be completed." This method is shown in Fig. 106(*b*) and is still in use.

Present Method of Procedure. The chief difficulty with this system is caused by the convergence of the meridians already noted in connection with azimuth traversing. In order to harmonize the various requirements of the acts passed by Congress relating to public lands, the following methods of procedure have been adopted by the General Land Office and are given in detail in the *Manual of Surveying Instructions for the Survey of the Public Lands of the U.S.*, issued by the Commissioner of the General Land Office. These instructions are issued for the direction of those engaged on public-land surveys.

Initial Points, Principal Meridian, and Base Line. As the first step, initial points are established astronomically under special instructions and from the initial point a *principal meridian* is laid out north and south. Through this initial point a *base line* also is laid out as a *parallel of latitude* running east and west. On the principal meridian and base lines, the ½-mile, 1-mile, and 6-mile corners are permanently located and, in addition, the points at the intersection of the line with all streams, lakes, or bayous prescribed to be meandered are located. These lines may be run with solar instruments, but their correctness should be checked by observations with the transit on Polaris at elongation.

Standard Parallels. Standard parallels, or correction lines, are run east and west from the principal meridian at intervals of 24 miles north and 24 miles south of the base line, and the law provides that "where standard parallels have been placed at intervals of 30 or 36 miles, regardless of existing instructions, and where gross irregularities require additional standard lines from which to initiate new or on which to close old surveys, an intermediate correction line should be established to which a local name may be given, and the same will be run, in all respects, like the regular standard parallels."

Guide Meridians. Guide meridians are extended *north* from the base line or standard parallels at intervals of 24 miles east and 24 miles west of the principal meridian.

This results in dividing the country into sections having the bottom and both sides 24 miles long but less than 24 miles at the top, owing to the convergence of the meridians, since the sides are run as true meridians, Fig. 107.

When conditions are such as to require the guide meridians to run *south* from a standard parallel, or correction line, they are initiated at properly established closing corners of the given parallel, that is, they are begun from the point on the parallel at which they would have met it if they had been run north from the next south parallel. This point is obtained from computation and is less than 24 miles from the next east or west meridian by the convergence of the meridians in 24 miles.

In case guide meridians have been improperly located too far apart, auxiliary meridians may be run from standard corners, and these may be designated by a local name.

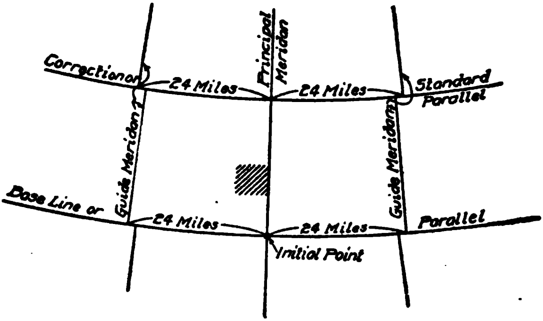
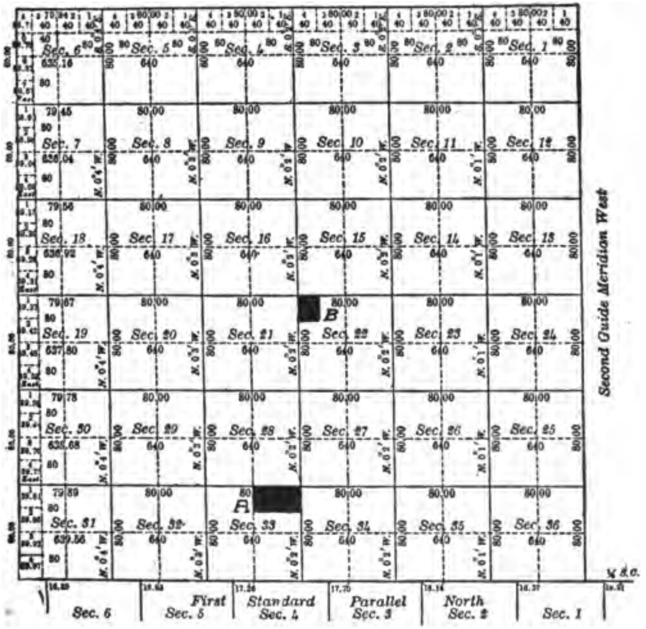


Fig. 107. Diagram of Surveys Showing Location of Townships

Townships. The next step is to subdivide each of these 24-mile squares into townships. This is done by running three intermediate north and south lines from which straight lines are run east and west to the proper points on the adjacent meridians. All the townships situated north or south of each other form a *range* and are designated by their number east or west of the principal meridian. Thus the first range west of the meridian would be designated as Range 1 West (R 1 W). Each *tier* of townships is designated by its number north or south of the base line, as Township 2 North (T 2 N). This township is shown cross-hatched in Fig. 107.

Sections and Quarter-Sections. The subdivision of each township into sections and quarter-sections is next accomplished. This is done in such a way as to throw the discrepancy, caused by the convergence of the meridians, into the most northerly and westerly quarter-quarter sections as shown in Fig. 108. This is in accordance with Section 3 of the act of Congress approved May 10, 1800, that "in all cases where the exterior lines of the townships, thus to be subdivided into sections and half-sections, shall exceed, or shall not extend 6 miles, the excess or deficiency shall be specially noted and added to or deducted from the west or north ranges of sections or half-sections in such township, according as the error may be in running lines from east to west or from south to north; the sections and half-sections bounded on the north

Township No. 6 North, Range No. 9 West, of a Principal Meridian



The above plot represents a theoretical township with perfect subdivisions, contiguous to the north side of a Standard Parallel; in assumed Latitude 43 15' N., and Longitude 100°00' W. of Gr. Area 23024.16 A.

Fig. 108. Typical Plat of Theoretical Township

and west lines of such townships shall be sold as containing only the quantity expressed in the returns and plats, respectively, and all others as containing the complete legal quantity."

Further Division of Public Lands. By various acts of Congress the division of public lands into quarter, half-quarter, and quarter-quarter sections is as follows:

Act approved, February 11, 1805, directs the subdivision of public lands into quarter-sections and provides that all the corners marked in the public surveys shall be established as the proper corners of sections or subdivisions of sections which they were intended to designate, and that corners of half- and quarter-sections *not marked* shall be placed, as nearly as possible, "equidistant from those two corners which stand on the same line." This act further provides that "the boundary lines actually run and marked . . . shall be established as the proper boundary lines of the sections or subdivisions for which they were intended; and the length of such lines as returned by . . . the surveyor . . . shall be held and considered as the true length thereof . . . ."

Act approved, April 24, 1820, provides for the sale of public lands in half-quarter sections and requires that "in every case of the division of a quarter-section the line for the division thereof shall run north and south."

Act approved, April 5, 1832, directed the subdivision of public lands into quarter-quarter sections and that in every case of the division of a half-quarter section, the dividing line should run east and west, and that fractional sections should be subdivided under rules and regulations prescribed by the Secretary of the Treasury.

On March 3, 1849, the Department of the Interior was created, and the act provided that "the Secretary of the Interior shall perform all the duties in relation to the General Land Office of supervision and appeal now discharged by the Secretary of the Treasury. . . ." By this act the General Land Office was transferred to the Department of the Interior, where it still remains.

Method of Describing Areas. Referring to Fig. 108, the shaded area *A* would be described as the north half of the northeast quarter of Section 33, Township 5 North, Range 9 West, Principal Meridian, or simply N ½ N E ¼ S 33 T 5 N R 9 W P M. The section *B* is N W ¼ N W ¼ S 22 T 5 N R 9 W P M.

Corners and Discrepancy. Corners are marked by trees, posts, stones, and earth mounds. Much of the earlier work of subdivision was very poorly done. The land was then of little value and correspondingly little care was exercised in the surveys. Several principles for the guidance of surveyors in resurveys have been established, but when individuals have bought the land, the corners as actually set must remain the corners, however erroneously placed.

CITY SURVEYS

Work of City Surveyor. The work of the city surveyor may be divided into two classes, namely, the laying-out of streets, the division of a section of land into blocks, and the subdivision of the blocks into lots; and surveys of city lots in sections already subdivided. In addition, he is also called upon to give the curb lines and grades for city streets. Attention has already been called to the fact that work of this character demands a much higher degree of accuracy than ordinary farm or land surveying.

Conditions in Accurate Taping. It is generally necessary to carry on the chaining with an accuracy of 1 in 10,000 or better, and, while temperature is neglected and the pull estimated so that the resulting stretch in the tape would about compensate for the sag in ordinary chaining, it is necessary in work of this kind to allow for temperature, sag, and pull. Many tapes have been devised with special adjustable scales, etc., to allow for these factors. For the ordinary steel tape, the correction may be computed from the formulas following.

Standard Tape. Another point which is very important is the matter of standards. An engineer in purchasing a tape for accurate work should ask the manufacturer for a certificate stating under what conditions it is standard. Usually the length is standard when the tape is supported throughout under a pull of 2 or 3 pounds and at a temperature of 62° F. At a slight increase in cost a tape can be obtained standard under a pull of 10 to 20 pounds, such as is required for field work, and in this way the stretch correction is eliminated. The U.S. Bureau of Standards, at Washington, D.C., will furnish a certificate, for a small fee, giving the corrections for any tape sent to them, and a surveyor engaged in work of this kind should have at least one standardized tape with which the tapes used in the field may be compared.

Correction for Temperature. A steel tape expands about 0.0000065 of its length for each degree Fahrenheit; hence, if the tape is standard at 62° F., the correction T to be applied to a measurement obtained at any other temperature t, for a length l, is

$$T=0.0000065\,l\,(t-62°) \qquad (1)$$

It will be noted that the correction is + when t is above 62° and − when below. The student should satisfy himself that this is correct, and in this connection should review the article on Correction of Errors in Chaining, "Plane Surveying", Part I. The correction for temperature amounts to 1 in 10,000 for a difference of about 15° from the standard temperature. When taping on a very sunny day, it is extremely difficult to get the exact temperature of a tape, and on days when the sun shines alternately because of clouds, it is almost impossible. For this reason, it is best to do the taping on overcast days or in the evening after sunset when the tape temperature is about that of the surrounding air.

Correction for Sag. In some cases it is possible to use the tape supported throughout, as for example, in city streets where it can rest on the curb or sidewalk; but usually it is necessary to support it at intervals and to allow it to sag between. The curve it takes is called the *catenary*, and the difference between the arc and the chord is

$$C=-\frac{l}{24}\left(\frac{Wl}{P}\right)^2 \qquad (2)$$

in which l is the unsupported length in feet; W is weight of tape in pounds per foot; and P is pull applied in pounds. Note that the correction is always subtracted from a measured distance to get the true distance.

Correction for Pull. A spring balance is generally used, by means of which a standard pull is put on the tape. If the tape is standard for a pull of P_0 pounds and is used with a pull of P pounds, the stretch correction is

$$S=(P-P_0)l\div(AE) \qquad (3)$$

in which l is as in (2); A is the area of cross-section of the tape in square inches; and E is the modulus of elasticity of the steel, which is about 29,000,000. Note that the sign of this correction follows the same law as that for temperature.

EXAMPLES FOR PRACTICE

1. A certain tape is standard at 62° F. when supported throughout under a pull of 2 pounds; its cross-section is 0.0034 square inch, and it weighs 0.0113 pound per hundred feet. A distance is recorded as 261.322 feet; the pull used is 20 pounds; the tape is supported at 100- and 200-foot marks; and the temperature is 82 degrees. Find the correct length.

Ans. $T=+0.033$; $C=[-(2\times0.013)+0.003]=-0.029$; $S=+0.048$; distance = 261.274 feet

2. A 100-foot steel tape is standard under the same conditions as above. Find the pull necessary to make the sag and stretch equal, assuming a temperature of 62° F. Ans. 19 pounds

3. How much should the pull be for a length of 50 feet?
Ans. 13 pounds

Subdivision Surveys and Computations. The first step consists in making an accurate farm survey and also a topographic survey of the property to be subdivided. The roads or streets are then laid out on the topographic map. In making this preliminary plan, the street layout should conform to the general plan of the city; in fact, it may be necessary to lay out a rectangular system along fixed lines, in which case no topographic map is needed. If, however, the matter of street lines is not fixed, the lines must be sketched in on the topographic map, bearing in mind the necessary connections with adjoining streets, the division of the property into blocks that, in turn, may be divided into lots of the best shape and size, and the location of the streets in such a way as to make the excavation a minimum, both for the street itself and for the necessary grading of each lot to bring it to a suitable grade. The student can quickly realize that this preliminary layout is by far the most important part of the work. Slight changes in streets or grade often result in changes in size, shape, or frontage of lots that greatly enhance their value. Care must also be taken to make the lots, which are next to the adjacent sections, front in such a way that when these sections are subdivided, these lots will not be poorly located.

The final scheme, which should be accepted as such only after sketching out and studying all possible schemes, must usually be approved by the town authorities, and the space occupied by streets is, in many cases, deeded over to the city as public property. The street boundaries are then accurately laid out as tangents or curves of known radii, and the dimensions of all sides, angles, and areas of each lot are computed, so a description of any lot may be written.

City-Lot Surveys. The city surveyor is often called upon to make surveys of lots in parts of the city that have been subdivided and built upon. These surveys may be divided into three kinds: title, or possession, survey; architect's survey; and building-line survey.

Title, or Possession, Survey. The title survey is a general survey of the property, properly locating it in the block and giving the location of all walls, buildings, fences, etc., with respect to the lot lines, so as to show if the building itself is properly located in the lot or if other buildings encroach on the lot, that is, are built over the lot lines. The purpose of the survey is to show the property "as in possession", and it is made in case of the transfer or sale of a piece of property to prove that the property is that called for in the deed and is not encroaching upon the street or other property and, in turn, is not being encroached upon. All title companies require such a survey in addition to a thorough legal search of the deeds of the property before they will insure the title to the property. They usually maintain a survey office for doing this work.

Architect's Survey. The architect's survey gives the location of the lot showing the lengths of all sides and angles. It also gives the street grades; the location and depth of gas, water, sewer service, etc.; the location of hydrants, lamp-posts, and catch basins; and any other features which might influence the work of the architect. In sections where the buildings are detached, contours showing the elevation of the surface of the lot also are given. Such a survey is needed by an architect who designs structures to go upon the property.

Building-Line Survey. The purpose of the building-line survey is to give, by marks, the lines for the walls of the buildings so the contractor may erect the building properly on the lot. These reference points in city work are usually chisel marks cut in the curb or on the walls of adjoining property so the contractor can stretch a chalk line between them and thus get his lines. Frequently, they are offset 1 or 2 feet from the exact line, so that a batter board has to be erected to hold the chalk line.

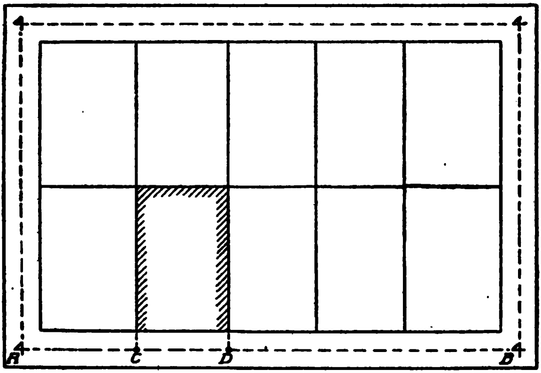


Fig. 109. Method of Making Survey of City Lot

It is also usual to make the elevation of the curb at the corners of the lot so that the contractor may use these levels in getting the proper elevation for the foundations and floors. For a detached building batter boards are usually set as described later.

Method of Making Survey of City Lot. Surveys of lots in city streets are made from a fixed system of reference lines. Usually the streets are marked by a system of monumentation established by the city. Monuments may not be placed at each street intersection, but the distances from the monuments, which may be stone monuments, marks cut in the top of manhole rims, etc., are given. These dimensions must be held as the true dimensions whether or not they check with the surveyor's tapings. To locate a lot, such as shown in Fig. 109, the city surveyor must first find the city monuments and traverse between them, dividing the difference between his taping and the distance shown of the city records (known as a *surplus* or a *deficiency*) proportionally between the blocks. His first work, therefore, is to locate the block lines from the city monuments; and, in order that he may not have to do this every time he makes a survey of a lot, he sets reference lines as shown at *A* and *B*, etc., Fig. 109, defined by chisel marks cut in the sidewalk, the lines being usually 10 feet from the building line. He is thus called upon to extend and complete the city system of monuments; in the larger cities, surveyors frequently keep parties in the field on work of this kind when they are not otherwise occupied. In this way a city surveyor accumulates maps and records that are of great value.

The next step in locating the lot shown, Fig. 109, is to determine the distances *AC* and *CD*. Now *CD* will equal the length of the block divided by the number of lots. It has been noted that the standard used by the city in taping for locating the city monuments may have been wrong or the work itself may have been in error. In the case considered, the block may be shown on the city map as consisting of 5 lots of 50 feet each, or a total length of 250 feet. It may actually measure 250.5 feet, in which case the surplus 0.5 is divided proportionately, making *CD* equal 50.1 feet and *AC* equal 50.1 plus the distance from the reference line to the lot line. A transit may then be set up at *C* and *D* and the angle of the lot side (90 degrees shown) turned off, thus locating the front corners of the lot. A traverse is then run through the building, if necessary, and the rear corners are located from the same reference line.

CONSTRUCTION SURVEYING

BUILDING AND OTHER CITY PROBLEMS

It would be impossible to give illustrations of all problems that occur in work of this kind, but the following examples will illustrate the methods usually followed.

Staking Out Building. The method usually followed in staking out a building is illustrated in Fig. 110(*a*). The excavation for the foundations would, of course, remove any stakes placed at the exact corners of the building; for example, a stake placed at *A* would be dug up and lost. For this reason, batter boards, Fig. 110(*b*), nailed to posts placed outside the limits of the construction are used to reference the corners.

The corners *A*, *B*, etc., Fig. 110(*a*), are first staked out. When the instrument is at *A*, it is sighted at *B*, and the batter boards are marked at *C* and *D*. An angle of 90 degrees is turned off and the corner *G* located; also the batter boards at *E* and *F* are marked, etc. When the stake at *A* is removed, the corner *A* can be recovered by stretching a line from *C* to *D* and another from *E* to *F*, the corner being at the intersection. The mason, in building the walls, also works from these lines. A similar procedure is followed in staking out other structures.

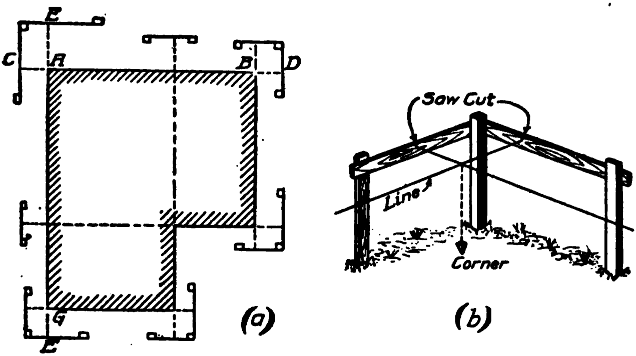


Fig. 110. Diagram Showing Method of Staking Out Building

Baseball Diamond. A baseball diamond is shown in Fig. 111. The shortest distance from the home plate to any fence between the foul lines should not be less than 235 feet. There should be 90 feet behind the home plate with a minimum of 50. The ground should be nearly level. The base lines should be level, and the fall from the pitcher's box to the home plate or base lines should not exceed 15 inches.

Tennis Court. A tennis court is shown in Fig. 112. The area required is about 50×120 feet. The net posts are 3.5 feet high and are placed 3 feet outside the side lines. The net should

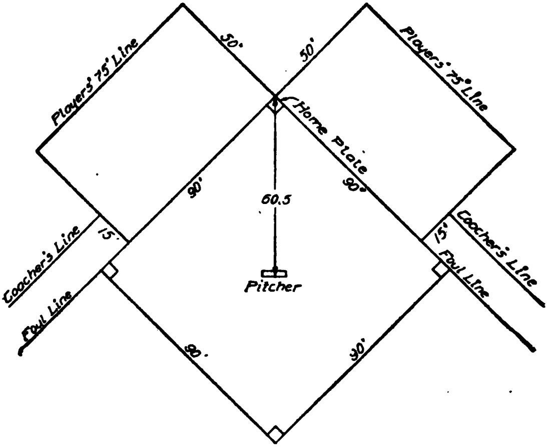


Fig. 111. Diagrammatic Layout of Baseball Diamond

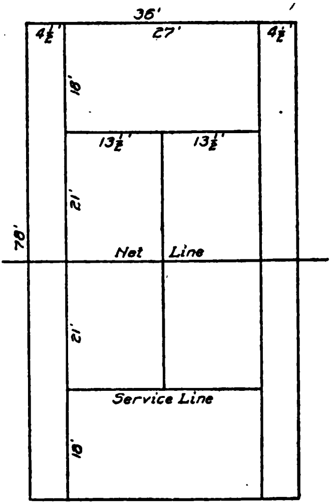


Fig. 112. Diagrammatic Layout of Tennis Court

be 3 feet high. The court should be given surface drainage by grading from the center toward the sides on a slope of about 1 in 200.

Street Grades. A transit and target rod are generally used for street grading. If the grade is straight between known points, such as intersections with other streets, the instrument may be set up at one of these points and the *H.I.* determined. This should not be confused with the *H.I.* used in leveling. In this case it is simply the distance of the horizontal axis (or wire) above the point over which we are set up. The target is set at this reading and held on the other fixed point. The telescope is then inclined, the horizontal wire is set on the target, and the vertical motion is clamped. The line of sight is now parallel to the given grade, and intermediate grade points may be obtained by *shooting-in,* that is, an intermediate stake may be driven down so that the horizontal wire cuts the target when the rod is held on the stake. In some cases stakes cannot be set to the exact height and a mark is placed on the stake at some even foot (say, 1 or 2 feet) above or below grade and the stake is marked $-1'$ or $+1'$, as the case may be. Marks are also placed on fences, curbs, or buildings, at

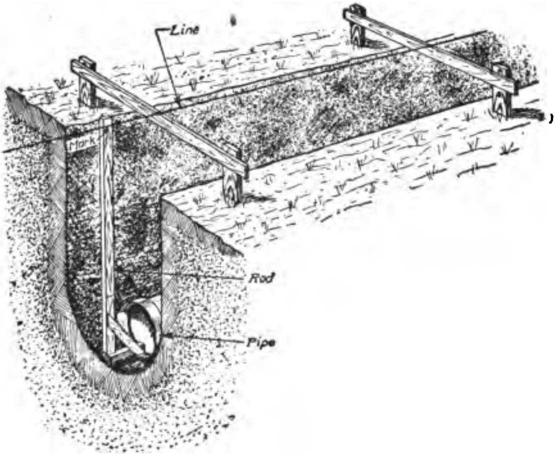


Fig. 113. Diagram Showing Method of Making Line and Grade for Sewers

convenient elevations, and marked $\overline{1\uparrow}$ or $\underline{2\downarrow}$, indicating that true grade is 1′ above or 2′ below the horizontal mark.

Sewers. One method of giving line and grade for sewers is shown in Fig. 113. Stakes are driven on each side of the trench at intervals of 25 to 50 feet, and batter boards are nailed across. A saw-cut in the board gives the line, and the height of the board is usually fixed so that the bottom of the saw-cut is an even number of feet above the sewer grade. The sewer grade is the grade of the invert, or inside bottom of pipe, and the pipe is adjusted to grade by means of a rod having a right-angle projection on the bottom, as shown in the illustration. The distance from the invert to the boards is marked on this rod, and intermediate sections of pipe are set by measuring down from a line stretched between the boards.

EXCAVATIONS

Classification. Two general types of excavation problems are common. First, where gravel, earth, etc., are to be taken out from a certain section and the volume so taken out is required. This is known as a *borrow pit*, but the method of computation can be used also for grading fields and other work of this character. The second type is work involving the taking of *cross-sections*, usually at intervals of 100 feet, such as is necessary in computing the excavation for road, railroad, canal, and similar work.

Borrow Pits. The area to be excavated or filled is first staked out in squares as shown in Fig. 114, the corners of each square being marked, and the base line A_0F_0, from which the squares are laid out, being outside the limits of excavation. A bench mark is then established, also outside the limits of the work, and the surface elevation is obtained at each stake or corner. After the work is completed the stakes will be covered up or removed, but the corners can again be located from the base line A_0F_0, and the new elevations obtained. The limit of the excavation or fill shown by the heavy irregular line should also be determined

By subtracting the original and final elevations at the corners, the cut or fill can be obtained. The cubic yards of earthwork for each square is then found by averaging the amount in feet of the cut or fill at each corner, multiplying by the area of the square, in square feet, and dividing by 27. The irregular portions, shown shaded, Fig. 114, are pyramids or wedges. For example, the section *abcd* is a wedge, and its volume can be found by averaging the area of the ends *ab* and *cd* and multiplying by the perpendicular distance *bd* between them. The section *cde* is a pyramid, the area *cde* being known, its volume equals this area multiplied by ⅓ the cut or fill at *d*, which is its altitude.

In finding the volume for the full squares, the following rule is sometimes used:

$$\text{cubic yards} = \frac{A\ (\text{sum } h_1 + 2 \text{ sum } h_2 + 3 \text{ sum } h_3 + 4 \text{ sum } h_4)}{4 \times 27}$$

in which sum h_1 stands for the sum of the cuts or fills at the corners that enter but one full square, that is, corners like B_1, B_4, D_5, E_5, etc. Sum h_2 stands for corners like C_1—which is used in computing the squares $B_1B_2C_2C_1$, $C_1C_2D_2D_1$, and $E_3E_4D_4D_3$, etc. The sum h_3 stands for corners such as D_2 and D_4, and sum h_4 for corners C_2, C_3, and D_3.

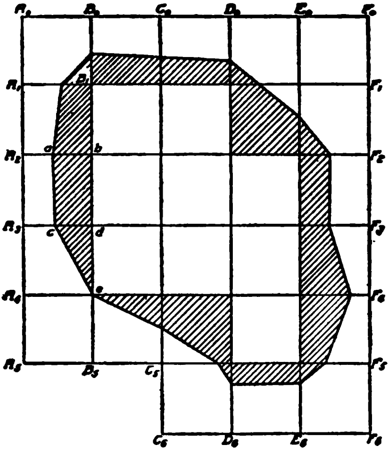


Fig. 114. Diagram Showing Borrow Pit Method of Handling Excavation

Examples 1. Suppose the cut in feet at the corners of 25-foot squares in Fig. 114 are as follows:

$B_1 = 1.2$, $B_2 = 1.5$ $C_1 = 0.8$, $C_2 = 2.3$ $D_1 = 0.8$, $D_2 = 2.0$ $E_2 = 1.2$, $E_3 = 1.9$
$B_3 = 1.1$, $B_4 = 0$ $C_3 = 2.6$, $C_4 = 1.8$ $D_3 = 3.6$, $D_4 = 2.1$ $E_4 = 2.1$, $E_5 = 0.7$
$D_5 = 0.6$

Find the volume of excavation in these squares, in cubic yards, by the above formula. Ans. 400 cubic yards

2. If *ab* equals 13 feet and *cd* equals 10 feet, find the volume in the wedge and pyramid. Ans. 7.1 and 1.7 cubic yards

Cross-Sectioning. One of the most important problems that confront the leveler is the setting of slope stakes, or cross-sectioning, from which may be determined the amount of earthwork in the cut or fill, and which marks the extreme limits of the excavation in building railways, highways, canals, irrigation ditches, etc. The details of this subject are fully treated in "Railroad Engineering", and only the fundamental principles will be taken up here.

Line on Level Ground. The problem is as follows: Given in Fig. 115 the required width of finished roadbed or channel *FD*, with proper side slopes—depending upon the kind of material—*FA* and *DB*, it is required to determine where these side slopes will intersect the natural surface of the ground with reference to the center line of the survey, that is, the location of *A* and *B* and the distances *AC* and *CB*. The center line is defined by a stake, carefully aligned and leveled, and a profile of it is prepared, on which the grade line is laid down, showing the elevation of the finished roadbed or channel *FD* with reference to the natural surface of the ground; that is, *CE*, the center cut or fill, as the case may be, is known.

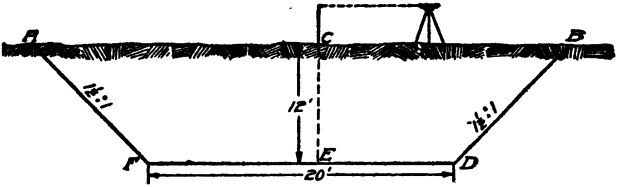


Fig. 115. Diagram for Excavation on Level Ground by Cross-Section Method (Drawing not to Scale)

Assume the ground to be level, transverse to the center line; depth of cut at center, 12 feet; side slopes, 1½ feet horizontal to 1 foot vertical; width of cut at bottom, 20 feet.

Set up the instrument in some convenient position that will command a view of as much ground as possible. Hold the rod on the ground at the center stake and note the reading. Suppose it to be 3.5 feet. If the ground is level, the distance from *C* to *B* is evidently $10+(12\times1\frac{1}{2})$, or 28 feet, and the rod should again read 3.5 feet when held at *B*. The point *A* would be found in the same way. The notes would be kept as shown in the following tabulation:

Station	Distance	Left	Center	Right		Area	Cubic Yards
175	50	$\frac{+12.0}{28}$	+12.0		$\frac{+12.0}{28}$		
176		$\frac{+3.0}{14.5}$	+ 6.0		$\frac{+9.2}{23.80}$		
176	50	$\frac{+2.5}{13.75}$	+ 5.0	$\frac{+7}{9}$	$\frac{+8.0}{22}$		

Line on Hillside. The preceding example illustrates one of the simplest cases that occur in practice. Take the case of a line located on the side of a hill, Fig. 116.

Depth to grade at center, 6 feet; width at bottom, 20 feet; side slopes, 1½ to 1. As before, hold the rod on the ground at *C* and determine the height of instrument above *C*. Suppose this to be 5.5 feet. If the ground were level through *C*, it would be necessary to measure to the right 10+(6×1½), or 19 feet to point *D* and the rod should read 5.5 feet. Instead it reads, say, 2.8 feet. We know therefore that we have not gone out far enough by (5.5−2.8)1½, or 4.05 feet, if the ground were level through point *D*, bringing us to point *E* where the rod should read 2.8 feet. Suppose it reads 2.3 feet. We must then go out 0.75 foot farther, each move bringing us closer to point *B*. This operation may be repeated as often as is considered necessary, but, with a little experience in this sort of work, the instrument man can direct the rod closely enough to point *B* for all practical purposes. We then enter the notes in the second line of the record shown above.

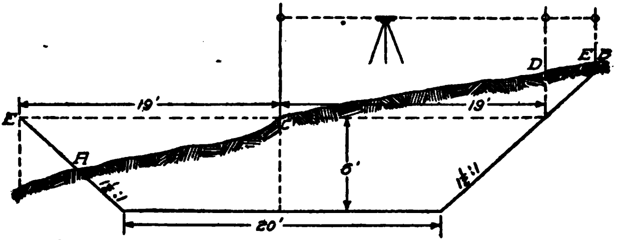


Fig. 116. Diagram for Excavation on Hillside by Cross-Section Method (Drawing not to Scale)

On the left of the center, these operations are reversed, that is, we measure out 19 feet and instead of the rod reading 5.5 feet, it reads, say, 8.5 feet. We know then that we are out too far by 4.5 feet. We then move in toward the center the required distance, read the rod again—noting how much it differs from 8.5 feet—and enter the final results in the notes.

Irregular Slopes. A third case is shown in Fig. 117, in which the transverse slope is not uniform. The method of procedure is the same as in the other cases, but the rod should be held at point *D* where the slope changes, in order to find its height above grade. Enter this and the distance out in the third line of the notes.

The transverse section may be very irregular, in which case it may be necessary to take readings at several points in order to calculate the area of the sections with more exactness. At times a section will be cut partly in rock and partly in earth, forming a compound section. Each material will, of course, have its own proper side slope, and the depth and extent of each must be determined by tests.

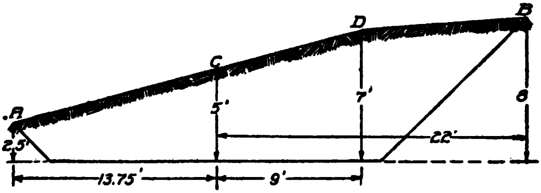


Fig. 117. Diagram for Excavation on Irregular Slope by Cross-Section Method (Drawing not to Scale)

Fills. In case the section is on fill instead of in cut, the method is the same as in the preceding cases and is illustrated in the following instances:

Take the case where the surface of the ground has a transverse slope, Fig. 118. Hold the rod at point *C* and suppose it reads 9.25 feet. If the ground were level through *C*, we would have to go out to the right 9+(6.25×1.5), or 18.4 feet to some point *D*. But there the rod reads, say, 1.5 feet; hence we know we are out too far by 7.75×1.5, or 11.63 feet, bringing us back to some point, as *E*, and the rod now reads 3.5 feet, and we move out again 2.0×1.5, or 3 feet. Therefore, we move back and forth until we find point *B*, where the computed rod reading and the actual reading agree and enter the result in the notebook as before, except it will be marked − instead of +.

Sometimes it will be found that a part of the section is in cut and a part on fill, but the methods outlined will serve in any case.

The distance between the sections longitudinally will depend upon the nature of the ground. On uniformly sloping or level ground they may be taken 100 feet apart. Over uneven ground it may be necessary to take them as close together as 25 feet or even less. In the sections, themselves, a sufficient number of rod readings should be taken so that the area of the sections may be determined with reasonable accuracy.

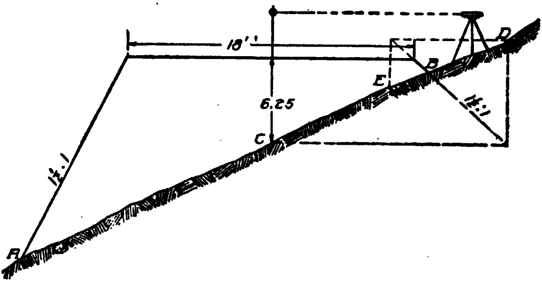
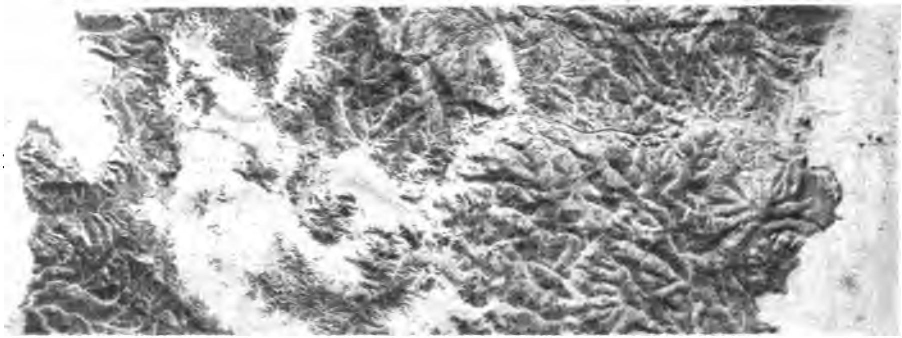


Fig. 118. Diagram for Fill on Slope by Cross-Section Method
(Drawing not to Scale)

After the work is completed, the notes are plotted, usually on cross-section paper, and the areas are determined by counting up the squares of the paper in the cross-section, or with a planimeter, or by some other method. The volume between adjacent cross-sections is usually found by taking the average of the areas of these sections and multiplying it by the distance between the sections. This is known as the *average end-area method.* It is not absolutely correct from a mathematical standpoint, and for greater accuracy the prismoidal formula is used. This is discussed in "Railroad Engineering", as already noted, where the problem of computing the excavation, in passing from cut to fill, is also considered.



RELIEF MAP OF PANAMA CANAL ZONE
Photograph by Howell's Microcosm, Washington, D. C.

PLANE SURVEYING

PART IV

TOPOGRAPHIC SURVEYING AND MAPPING

INTRODUCTION

Definition of Topography. Topography is the art of making a graphic representation of, namely, the *hydrography*, or the outlines of all water courses and bodies of water; the *relief*, or the form of the surface of the ground; and the *culture*, or all the artificial features and structures built by man. In general, the art consists in representing all these features in such a way that they are easily intelligible from the map, while economy requires that the methods employed shall be such as to make a map which shall give all the necessary information with the minimum of work in surveying and drafting.

Scale of Map. One of the first things to be decided is the scale of the map to be used, for on this decision will depend the character of the work which is to be done. For example, if the map is to be plotted on a small scale, it will be useless to do the surveying work with a closeness of detail which it will be impracticable to represent on such a small scale. The scale of the map must depend on the use which is to be made of it, as will be discussed later. On the one hand, a very small scale will mean that desired details cannot readily be shown or else can be shown only by the draftsman using the most extreme care; on the other hand, a large scale will mean that if the survey covers a considerable area of ground, the map will be inconveniently large. In brief, the true criterion for the scale of the map is that it should be drawn to the smallest scale which will properly represent the topographic features which must be accurately shown on the required map.

METHOD OF FIELD WORK

Choice of Method. The choice in the method of field work depends upon the same considerations that govern the required scale of the map. The method adopted should have sufficient accuracy to properly represent all required features; on the other hand, time and effort should not be wasted in plotting minute details with painstaking accuracy when it is known by experience that such accurate work will be of no practical use.

Reconnoissance Surveys. When an engineer is making an exploratory survey—for a railroad, for example—through a comparatively unknown tract of country, he may determine elevations with a barometer; the directions of the lines with a hand compass; the slope of lines with a hand level, or clinometer; and distances with a pedometer, or pace measurer, or with an odometer, an instrument which records the revolutions of a wheel of known circumference and which is rolled over the route traversed. These crude methods have the merit of great rapidity and cheapness and can be made with sufficient accuracy to determine whether that general route should be immediately rejected or whether it deserves further consideration and a more accurate survey.

When a topographic survey of a large area is to be made regardless of the method of utilization, such a reconnoissance survey is sometimes made for the purpose of planning the detailed work, selecting triangulation stations, if the triangulation method is to be used, and determining the method best suited for the results to be accomplished.

Accurate Surveys. To pass to the other extreme, topographic surveys, which are to be used as a basis for locating buildings, bridges, etc., must be accurately made to a large scale and must show every detail. This, in general, requires a transit and a stadia traverse, or the contours may be obtained by the square method described later.

Details. The representation of details depends upon the scale of the map. Small scale maps are used to represent a very large area of country. Usually, in these maps, nothing is indicated except the most important features. The outlines of the largest bodies of water, such as oceans, lakes, and rivers, are represented as closely as is practicable. Important cities are represented by arbitrary signs, such as dots and small circles. For special purposes, such as for railroad maps, the railroad is represented in a very exaggerated way; but, on the other hand, as the scale of the map grows larger, more and more of the details can be represented until, on the larger scale maps, even the nature of the crops in individual fields can be represented by appropriate topographical signs. When the relief of the ground is to be shown, and especially if it is of importance to represent it with accuracy, the usual method is to show it by contours. Since these lines are liable to be mistaken for other lines, such as roads, rivers, etc., the device of employing different colors for the various topographical signs is employed. If it were practicable to take, from a balloon at a sufficient height, a photograph of the entire area of country to be surveyed, it would give what might be considered in some respects an ideal representation of the area to be surveyed. But such a picture would be useless in many respects; many small but very important features would be almost indistinguishable. The essence of proper plotting consists in representing, perhaps in an exaggerated form, those features which it is especially desired to show on the map and in omitting other details which are of no practical importance, at least, for the particular purpose for which that map is made.. The method of plotting contours also permits an accurate representation of differences of elevation which would be unobtainable from a balloon picture.

Since the final result of a topographic survey is the map, and since the method of surveying to be used depends upon the scale of the map and the nature of the physical characteristics which are to be represented on the map, the various methods of mapping will be described first.

METHODS OF MAPPING

Scales Used. Topographic maps are frequently made as the basis on which to design constructive work, such as buildings, factories, dams, reservoirs, sewage and waterwork systems, canals, railroads, etc. For each case there are certain topographical details which must be represented and a certain accuracy with which they must be measured and plotted, but a scale of 100 feet per inch is usually large enough, although, when the details are to be given with great precision, a scale of 50 or even 10 feet per inch is used. Using a scale of 100 feet per inch, every building can be drawn in its actual form and to within a foot of its actual dimensions, and even the separate rails of a railroad track can be clearly indicated. With such a scale, a map 30 inches square, including the border, will show about ½ mile each way, or ¼ square mile in area. This is sufficient for many purposes, and the scale is therefore largely used.

Sometimes there is a great advantage in having maps on a scale so small that a very large area may be shown on a single map of convenient size. On such maps, although the general locations of buildings, etc., are desired, their exact shape or size is of comparatively little importance. The U.S. Geological Survey indicates residences, for example, on maps of the scale of approximately 1 mile per inch, Plate IV, by minute squares of solid black which, of course, do not represent the houses either in shape or size. As the area to be represented grows larger, this requirement must be met in one of the following ways, viz, by increasing the size of the map until it approaches the limit of convenient size for handling; by making sectional maps, which for some uses are impracticable; or by decreasing the scale. When the scale is decreased, it is possible to show the same data by plotting the work finer, but this generally leads to a map that has a very flat appearance, on which the main features do not stand out. The most practicable methods of decreasing the scale are by judiciously eliminating details which will have little or no value for the immediate purpose for which the map is made or by modifying the conventional signs employed in such a way that they will convey sufficient information without sacrificing simplicity and clearness. As we decrease the amount of matter placed on a map, we increase the clearness and ease of distinguishing those things which are shown. While the inexperienced engineer should be very cautious to avoid the omission of details which might prove useful or essential, the experienced engineer will save time in the field and improve the character of his map by omitting details which his experience tells him will certainly not be needed in the work for which the map is to be used. When an engineer is making a map for a special purpose, such as railroad work, he has no justification in crowding the map with details which might be of importance in some other kind of work but which will not, under any conditions, be of importance in the contemplated railroad construction.

Contours. A contour is a line joining points on the earth's surface which have the same elevation. The simplest method of appreciating the nature and location of contours is to consider them as the shore lines formed by the ocean if its surface were to be raised successively to the different levels assumed for each contour. This is illustrated in Fig. 119. Suppose a right cone, Fig. 119(*a*), is 6 feet high and is placed in a tank. When the water just covers the bottom of the tank, the contour is the outside circle shown at the top of Fig. 119(*a*), and as the water is raised by amounts of 1 foot, the successive shore lines, or contours, are shown by concentric circles. If the cone is inclined, the contours will be as shown in Fig. 119(*b*). Note that the contours, when evenly spaced, as at *A*, indicate a uniform slope; when close together, as at *B*, they indicate a steep slope; and when far apart, as at *C*, they indicate a gentle slope.

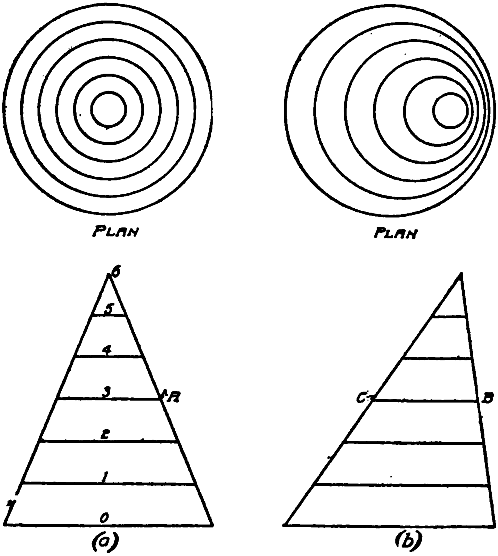


Fig. 119. Plan and Elevation of Right and Inclined Cones, Showing Method of Representing Contours

Fig. 120 will also aid the student in visualizing contours. If a number of copies of a contour map, printed on cardboard having a scale thickness equal to the contour interval, are cut out, each along a certain contour, a model such as is shown in Fig. 120 is obtained when these sections are placed one over the other. By filling up the contour intervals of this model with plaster or clay a *relief map* of the ground surveyed may be obtained.

Contour Interval. A certain uniform vertical increment, or contour interval, is always taken in making a map, the intervals varying with the scale of the map and the character of the country. For the closest of detail work on a larger scale, such as might be used for landscape gardening, a contour interval of 1 or 2 feet might be necessary. But for ordinary purposes 5 feet or more is used as the contour interval. The topography required for preliminary railroad surveys is usually done with a 5-foot contour interval. The topographic surveys made by the U.S. Geological Survey which are plotted at a scale of 1 or 2 miles per inch, approximately, are made with a contour interval varying from 20 to 100 feet, the 100-foot interval being necessary where steep mountain ranges are to be represented.

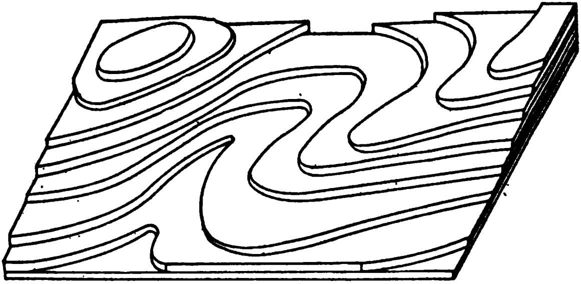


Fig. 120. Typical Contour Model

Principles Applied to Contours. There are certain principles regarding contours which the student must keep in mind and by which he may avoid errors in doing this work. Barring a few exceptional cases which are mentioned later, the following principles may be laid down: (1) a contour is always a continuous line inclosing an area; (2) since that area may and frequently does run off the map, the contour may run from one edge of the map to any other point on the edge; (3) a contour line is always a continuous line—it never stops abruptly; and (4) contour lines do not cross each other or merge into each other. The exceptions to the above rules occur only where the contours run into an artificial vertical wall or a precipice which is actually vertical. In the very unusual case of an overhanging cliff the contours might actually cross each other very slightly, but with exceptions which are readily recognized when they occur, the above principles may be considered as true, and when the principles are violated on a map, it may be considered as evidence of incorrect work.

Contours are shown in the same manner on maps of large or small scale; the interval alone varies. To prevent confusion, the contours are usually drawn with brown or brick-red ink. The contour elevations are always given with reference to some datum plane, such as sea level, and the elevations are always a multiple of the contour interval. For example, if the contour interval is 20 feet and the datum plane is sea level, then each contour elevation will be a multiple of 20 even though the whole tract is far above sea level. The contours might be at elevations 660, 680, 700, 740, etc. Then the 600-, 700-, or 800-foot contours would be made extra heavy and could be more readily distinguished even in steep places where the contours would be very close together.

The contours should be numbered at frequent intervals, so that the elevation of a contour at any point may be determined by following it for a very short distance to the nearest "ladder" of numbers. The student is referred to Plates II, III, and IV for illustrations of this point as well as of many others. In connection with contour work covering large areas, "The Interpretation of Topographic Maps", *Professional Paper No. 60* of the U.S. Geological Survey should be studied. Students intending to take up work of this character should read this publication.

Hachures. The representation of the relief of a country by hachures has been almost entirely superseded by the above described method of contours. Hachures are objectionable for the following reasons: they are difficult to draw neatly, are not so accurate as contours, and cover up the map to the exclusion

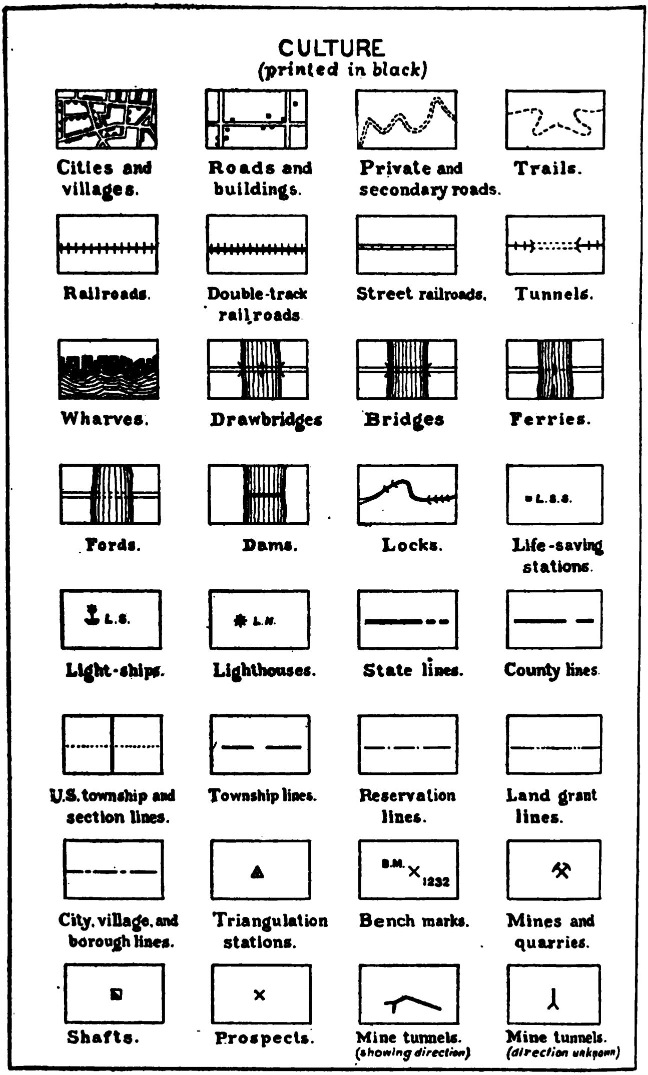
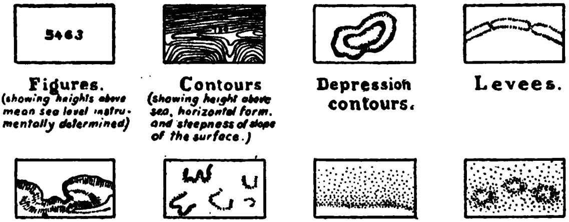


Plate II. Topographic Conventional Signs. For Illustrations of These Signs in True Colors, See Plate VIII

RELIEF

(printed in brown)



Figures. *(showing heights above mean sea level instrumentally determined)*

Contours *(showing height above sea, horizontal form, and steepness of slope of the surface.)*

Depression contours.

Levees.

Cliffs.

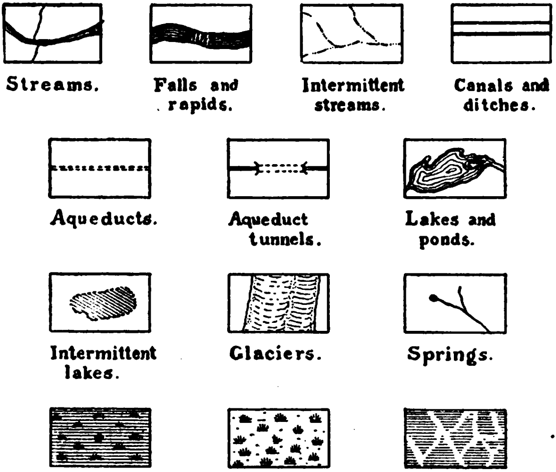
Mine dumps.

Sand.

Sand dunes.

WATER

(printed in blue)



Streams.

Falls and rapids.

Intermittent streams.

Canals and ditches.

Aqueducts.

Aqueduct tunnels.

Lakes and ponds.

Intermittent lakes.

Glaciers.

Springs.

Salt marshes.

Fresh marshes.

Tidal flats.

Plate III. Topographic Conventional Signs For Illustrations of These Signs in True Colors, See Plate VIII

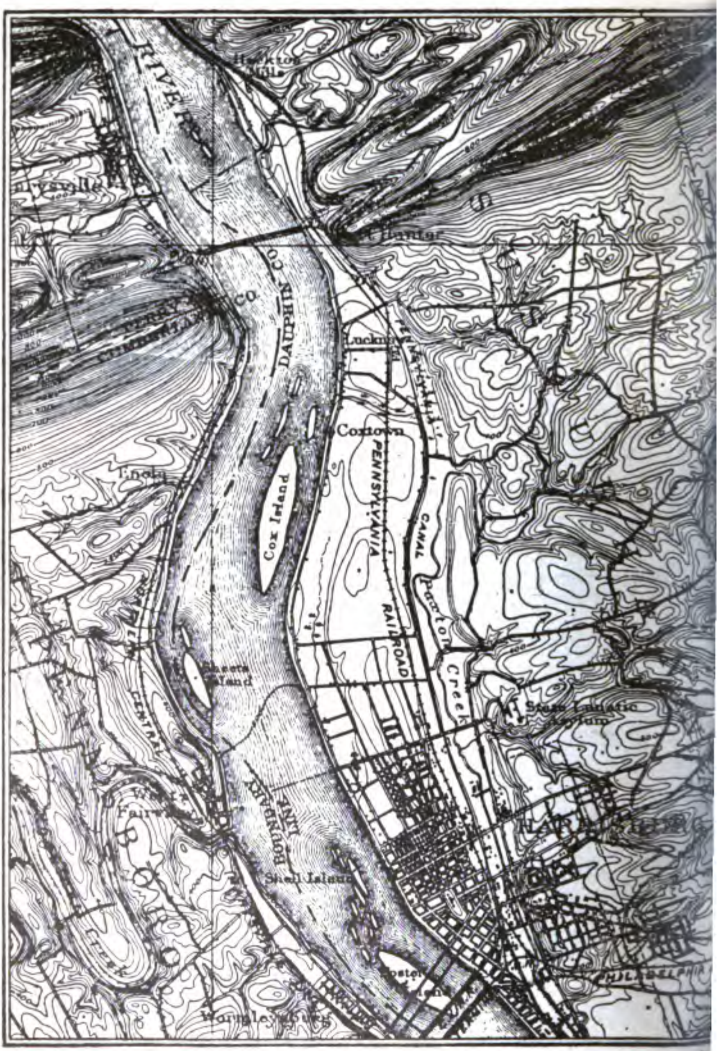


Plate IV. Part of Harrisburg, Pennsylvania, Sheet. U.S. Geological Survey

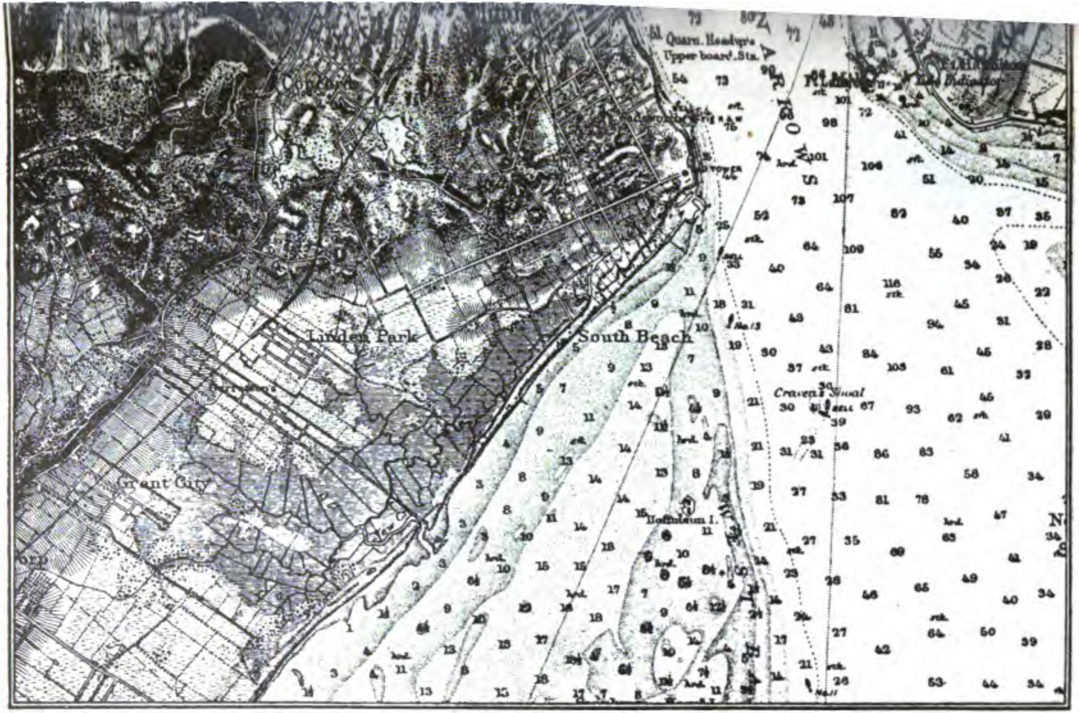
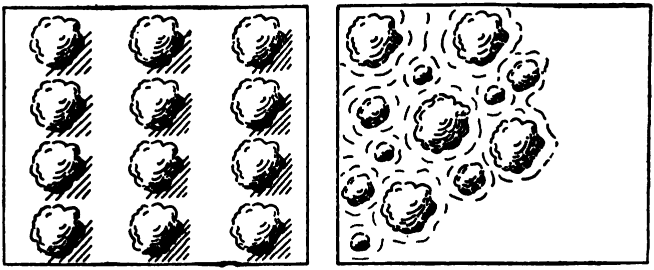
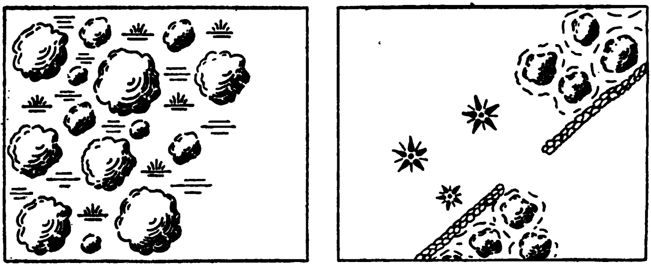


Plate V. Part of New York Harbor. Relief Shown by Hachures. U.S. Coast Survey



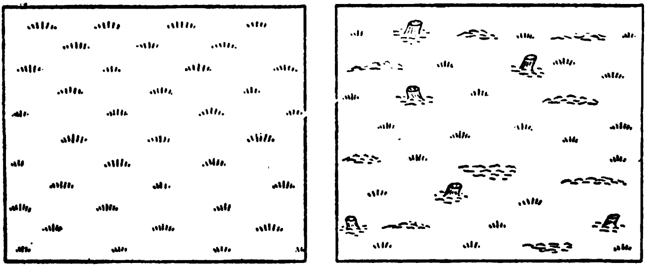
ORCHARD

WOODS



SWAMPY WOODS

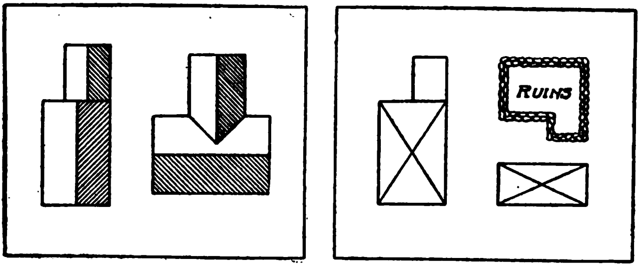
PINE TREES, BRUSH, & STONE WALL



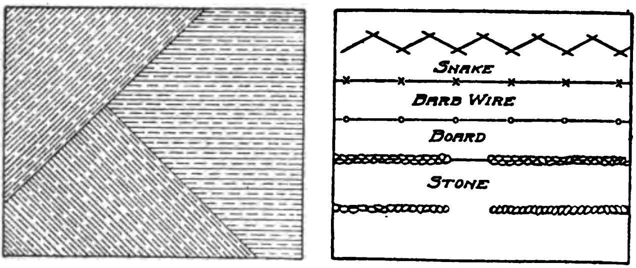
GRASS

PASTURE

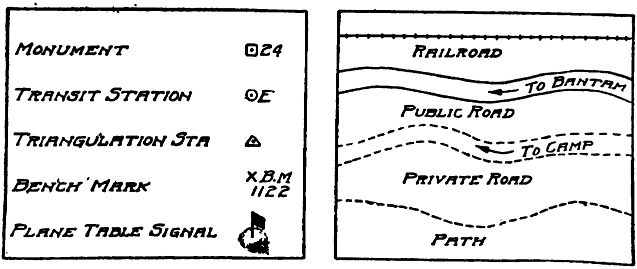
Plate VI. Conventional Signs for Scale 1″ = 100′. For Illustrations of These Signs in True Colors, See Plate VIII



HOUSES BARNS & OUTHOUSES



CULTIVATED FIELDS FENCES



SIGNALS, ETC. ROADS

Plate VII. Conventional Signs for Scale 1″ = 100′. For Illustrations of These Signs in True Colors, See Plate VIII

of other topographic details. Their principal use is to give a general idea of the character of the country when the survey has not been made in sufficient detail to render possible an accurate representation of contours. Hachures can be used to give a representation of mountain peaks, gorges, etc., whose forms are determined only by very approximate sketching when the observer is possibly 1 or 2 miles away from them, and absolutely no instrumental work is resorted to in determining their elevation.

In order to construct hachures with an approximation of accuracy, it is desirable to sketch in contour lines with a pencil at equal vertical intervals. The hachures are lines which are assumed to be perpendicular to a contour line at every point of their length or, geometrically, they represent in each case and at every point of their length the steepest line at that portion of the surface. The lines should be perpendicular to the contour lines and also perpendicular to any intermediate contour which might be interpolated between the contours which are drawn. This means that the hachures should be more or less curved, Fig. 121. The steepness of the slope, therefore, will be indicated by the length of the separate hachure lines, but the steepness is also indicated still more graphically by increasing the thickness of the hachures on the steeper slopes. Hachures have the advantage of calling attention to the steepness of slopes, which may be of extreme importance on military maps where accuracy is by no means essential, but on which the fact of a steep slope, even if it is no more than a bank of earth, is of importance. Hachures may thus be used to represent an artificial or a natural embankment when there is no other representation on the map showing any relief features of the ground.

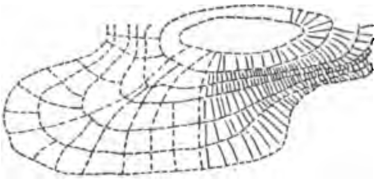


Fig. 121 Typical Sketch Showing Hachures, Dotted Lines Indicating Construction Lines Eventually Erased

Conventional Signs. *Small Scale Maps.* The conventional signs used by the U.S. Geological Survey for maps drawn to a scale of about 1 inch to the mile are shown in Plates II and III. A very complete book, *Topographic Instructions of the U.S.*

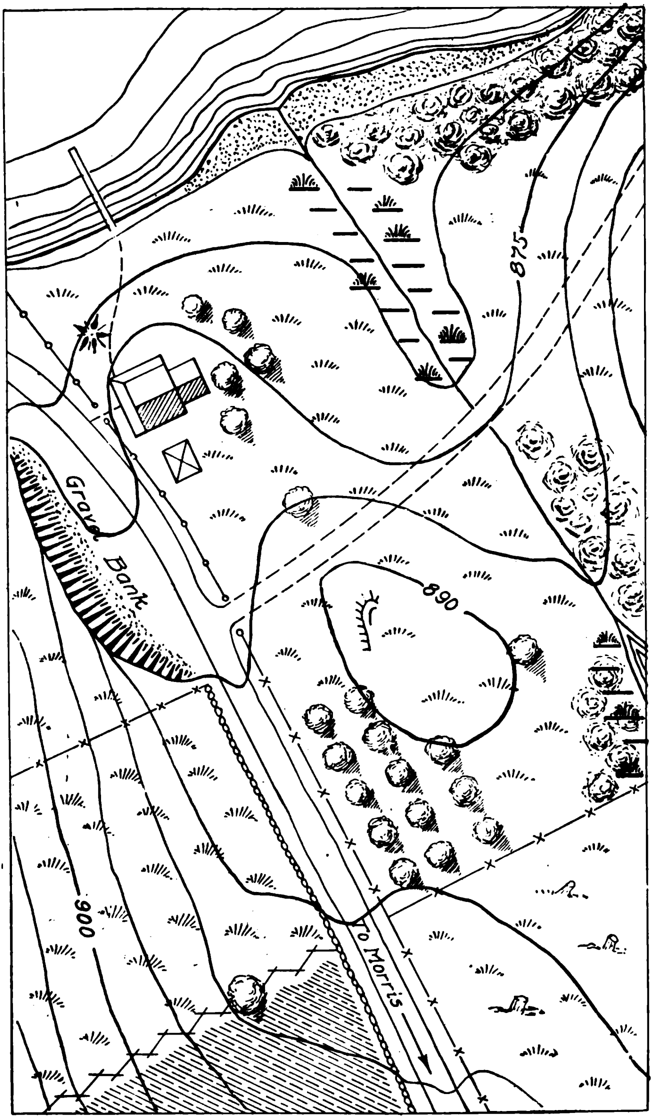


Plate VIII. Part of a Topographic Map.

Geological Survey, may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D.C., at 35 cents per copy, which gives, in addition to the instructions, a very complete set of topographic signs. Plate IV shows part of one of the maps of the Geological Survey in which these signs are used. In the government maps relief is shown in brown, hydrography in blue, and culture in black, and different styles of lettering are employed for names of towns, streams, railroads, etc. For color scheme, see Plate VIII. Plate V shows a section of some of the work of the U.S. Coast Survey.

Large Scale Maps. It is very seldom that the engineer in actual practice has to make maps of such large areas as those represented in Plates IV and V. In most cases his work covers only a small area and much more cultural detail is shown; also, it is necessary to show vegetation. Therefore, in addition to the signs shown in Plates II and III, he uses signs such as those shown in Plates VI and VII, and he is frequently called upon to invent additional signs to represent special features that are to be shown on the map. The student should practice drawing these signs both on paper and on tracing cloth. Use a No. 303 Gillott pen. Note that extremely fine lines are to be avoided and that there is desired a certain freedom of style resulting in a map which is clear and neat and does not look extremely complicated or labored. Fancy work, for the topographic details, border, or lettering, is always objectionable.

Plate VIII shows a section of topography drawn at a scale of 1″=100′, giving all signs in their proper colors and size. Note that buildings, roads, etc., are shown to scale while trees are not. It would be impossible to draw trees to exact scale as they often have a spread that would cover a large portion of the map, hence they are shown much smaller and only their relative size is indicated.

TOPOGRAPHIC SURVEYING

General Methods. Two general methods are used for topographic work, viz, the transit-and-stadia method and the plane-table method. In addition, several other methods are used when contours only are required. These are described first.

DETERMINATION OF CONTOURS ONLY

Square Method of Determining Contours. The square method of determining contours consists in establishing a gridiron of squares or rectangles over the desired area and in determining the elevation of each corner by means of a wye or dumpy level. The points where the contours cross the sides of the squares are next determined, usually with a hand level, Fig. 52, "Plane Surveying", Part I, and metallic tape. Thus, suppose one corner, Fig. 122, is at elevation 803.2 feet and the next corner, 812.5 feet. If a 3-foot contour interval is being used, three contours will cross the line between these corners, viz, 804, 807, and 810. The hand level is used to take a + sight of, say, 5.1 feet on the lower corner. The elevation of the line of sight is then 803.2+5.1, or 808.3, and a − sight of 808.3−807.0, or 1.3 feet, is required to locate a point on the 807 contour. The rod is therefore moved along the line between the corners until it gives a reading of 1.3 feet, and the contour is located by taking the distance, 36 feet, between the rod and the corner.

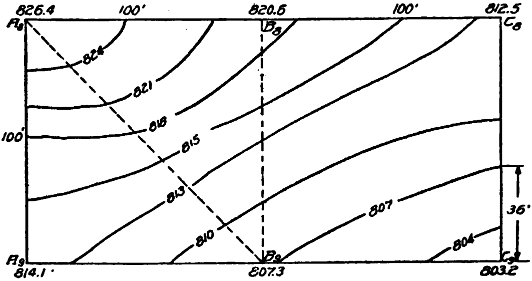


Fig. 122. Sketch Showing Square Method of Contours

In many cases it is not necessary to locate the points where the contours cross the sides of each square, as they may run practically straight for several squares. In other cases, such as that shown in the left-hand square, Fig. 122, bends occur at points inside the squares, and careful work requires that the contours be located on the diagonal line shown dotted. Careful sketches made approximately to scale in the field are essential.

The square method is probably the most accurate method of contouring and is used where a small interval, say, 2 or 3 feet, is required. Such a small interval is used in maps for landscape gardening and similar work and requires a corresponding increase in the accuracy of location.

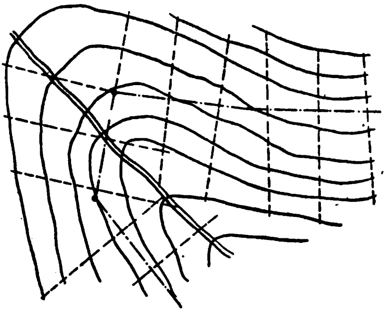


Fig. 123. Sketch Showing Cross-Section Method of Contours

Example. The following field notes were taken for the square B_8 B_9 C_9 C_8, Fig. 122. Study over the forms of these notes and draw the contours to a scale of 1″=25′.

From Sta.	Elev.	+S	H.I.	Contour	−S	To Sta.	Dist.
C_9	803.2	5.1	808.3	804	4.3	C_8	10.6′
				807	1.3	C_8	36.0′
				804	4.3	B_9	16.5′
C_8	812.5	6.2	818.7	810	8.7	C_9	44.0′
				813	5.7	B_8	4.0′
				816	2.7	B_8	38.0′
B_8	820.6	2.1	822.7	819	3.7	C_8	11.3′
				819	3.7	B_9	13.1′
				816	6.7	B_9	30.0′
				813	9.7	B_9	60.0′
B_9	807.3	3.1	810.4	810	0.4	B_8	15.2′
				807	3.4	C_9	3.1′

Cross-Section Method of Determining Contours. A traverse consisting of straight lines between instrument stations is run with a transit or other angle measurer and a tape. At certain intervals, usually 100 feet, stakes are placed and the elevations found with a wye or dumpy level. Cross lines are then run at right angles to the traverse lines as shown in Fig. 123. The elevation of the 100-foot stations being known from them, the contour points on the right-angle lines are determined in exactly the same way as in the square method.

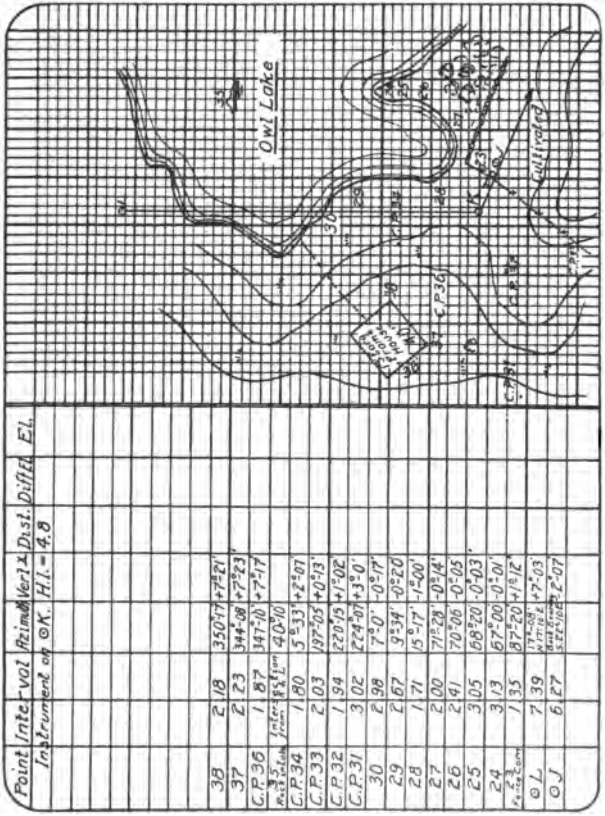
The cross-section method is used only when contours are to be determined over a long but narrow strip, such as is the case in a preliminary survey for a railroad, road, or canal. The usual interval in work of this kind is 5 feet.

TOPOGRAPHY BY TRANSIT AND STADIA

Method. The theory of stadia has already been explained. It is used principally for topographic work. The method used in work of this kind consists in running an azimuth traverse with a transit, measuring the sides by stadia, and taking side shots from each station of the traverse to locate topographic features. This method is illustrated in Fig. 124 which shows the proper form of field notes, the measurements to go on the left-hand page and the sketch on the right-hand page.

In running the transit-and-stadia traverse, the stations are selected so as to give commanding set-ups for the instrument from which all the necessary topographic details will be visible. Unlike a chain traverse, no attention need be paid to having the traverse sides run over suitable ground for chaining, as stadia work requires simply a clear sight to the rod. A closed traverse should always be used.

The transit is set up over station *A* and the plates are set at zero. The compass needle is then released and the instrument is turned until the needle reads zero when the lower motion is clamped. This procedure gives us as our zero azimuth either magnetic or true north, depending upon whether or not the transit compass has a declination arc, which is ordinarily close enough to the true direction, as it is only necessary to draw a pointer, Fig. 147, on the map. The next station *B* of the traverse is then sighted, and the azimuth, bearing (as read by the compass), the stadia interval, and the vertical angle are noted. It will also save a set-up if the last station (say, *Z*) of the traverse is also set and observed in the same manner. When the instrument is taken to station *B*, the azimuth is carried on in the manner already described for such work, that is, using the lower motion and sighting back at *A* with the plates set at the azimuth of *A* to *B* and the telescope transited. The magnetic back bearing also is read as a check (see notes ⊙*J* and ⊙*L*, Fig. 124) as well as the stadia interval and the vertical angle. The traverse is finally closed, and when the instrument is at *Z*, the azimuth *ZA* should check that determined from *A* within, say, $30''\sqrt{\text{number of stations}}$. The horizontal distances and differences in elevation are computed from the tables already given, using the average of the forward



Point	Interval	Azimuth	Vert ∠	Dist.	Diff El.	El.
	Instrument on ⊙K.		H.I. = 4.8			
38	2.18	350°-17'	+7°-21'			
37	2.23	344°-08'	+7°-23'			
C.P. 36	1.87	347°-10'	+7°-17'			
35 Rock in Lake	Intersection from K & L	40°-10'				
C.P. 34	1.80	5°-33'	+2°-07'			
C.P. 33	2.03	197°-05'	+0°-13'			
C.P. 32	1.94	220°-15'	+1°-02'			
C.P. 31	3.02	224°-07'	+3°-0'			
30	2.98	7°-0'	-0°-17'			
29	2.67	9°-34'	-0°-20'			
28	1.71	15°-17'	-1°-00'			
27	2.00	71°-28'	-0°-14'			
26	2.41	70°-06'	-0°-05'			
25	3.05	68°-20'	-0°-03'			
24	3.13	67°-00'	-0°-01'			
23 Fence Corner	1.35	87°-20'	+1°-12'			
⊙L	7.39	17°-08' N 17°-16' E	+7°-03'			
⊙J	6.27	Back Bearing S 22°-10' E	-2°-07'			

Fig. 124. Representation of Two Typical Pages of Field Notes, Showing Standard Form

and back readings and entering the values in the columns left for that purpose in the notes, Fig. 124. The error of closure is then computed; it should be about 1 in 1000 to 1500 for good work; if satisfactory, the latitude and longitude differences are balanced and co-ordinates computed and used for plotting.

Elevation of Each Station. The elevation of station *A* is either known or assumed and, using the differences in elevation for each course, the elevations of each station forward may be determined. Also the elevation of *A* will be again determined from *Z*, thus checking and closing the traverse vertically. The elevation of *A* thus determined will, of course, fail to check exactly. If it checks within, say, $0.05\sqrt{\text{number of stations}}$, it may be accepted as good work, and the closing error distributed equally for each course so as to make the vertical traverse close exactly.

Taking Stadia Shots. In taking stadia shots with the transit, the instrumentman must exercise constant watchfulness to avoid blunders. After setting up at a station, first determine the *H.I.*, then, when sighting a rod, read in the following manner:

(1) With the lower motion clamped in proper azimuth, loosen the upper motion and sight on the rod.
(2) Turn the telescope vertically until the middle wire cuts the rod at about *H.I.*
(3) Note what foot mark on the rod is near the lower stadia wire and turn the vertical motion until the wire is on this foot mark.
(4) Count up the feet, tenths, and hundredths to the upper stadia wire on the rod. This is the stadia interval. Call it out to the notekeeper.
(5) Set the middle wire exactly on *H.I.*
(6) Wave O.K. to rodman.
(7) Read the azimuth.
(8) Read the vertical angle.

This procedure releases the rodman as soon as possible so he can move on to the next point. It is also the simplest way to measure the stadia interval and is sufficiently accurate, although the middle wire should, theoretically, be exactly on the *H.I.* when the interval is read, Fig. 84, "Plane Surveying", Part II.

In reading side shots, it is not necessary for a skillful instrumentman to use the verniers. For vertical angles, the nearest 2′ or 3′ is close enough, and as azimuths are used only in plotting, 5′ is sufficiently accurate. When set up at any station, first get the back sight, then read to the next station in advance and have the rodman set a range pole at this station, so that when the side shots have been completed, the instrumentman may again sight this station and again read the azimuth, thus proving that the plates have not been moved by slipping or using the wrong motion.

Side Shots. As shown in Fig. 124, side shots are taken to located corners of houses as *37* and *38*, the dimensions of the house being paced by the rodman and recorded on the sketch. Fences, shore line, etc., are located. No boat being available, the rock *35* was located by intersections, reading an azimuth to it from stations *K* and *L*. These points are numbered on the sketch and in the notes. When the computations have been made, not only the direction and distance to each point have been obtained, so that it may easily be plotted on the map, but also its elevation. In many cases these elevations are sufficient to give us the contours; in other cases shots must be taken to certain points that are needed only for drawing the contours (marked *c.p.*, or *contour point*). The selection of these points requires much skill. Two things are required for properly locating contours: vertical control and horizontal control. Referring to Fig. 125, points *A*, *B*, and *C* must be located, as the slope *breaks* at these points. If we know the elevation of *A* and *B* and the slope is uniform between them, the distance of any point of required elevation from *B* may be found by direct proportion. But not only does the slope vary, requiring the location of points at the breaks, if these have not already been taken as topography shots, but the contours curve around hills and valleys, requiring a number of such points scattered over the area to properly give the bends, Fig. 124.

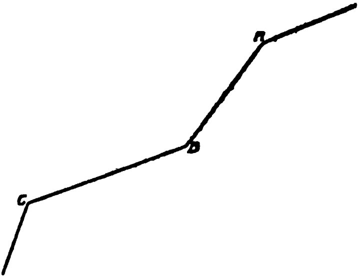


Fig. 125. Simple Profile Showing Method of Selecting Contour Points

The plotting up of the contours and the details for the final map are given later.

Reduction of Stadia Readings. For the traverse sides it is probably best to use Table III, given in "Plane Surveying", Part II. For side shots, of which there will be a large number in any survey, an easier method is desirable. Special slide rules are made for this purpose, but a diagram, such as that shown in Fig. 126, is as rapid as any method and is sufficiently accurate.

For side shots, it is sufficiently accurate to take $\frac{f}{i}$ as 100 and $f+c$ as 1. The diagram gives in one operation the difference in elevation and correction to be *subtracted* from 100 times the stadia interval to get the horizontal distance. Thus take the reading for point *31*, Fig. 124. One hundred times the stadia interval is 302, and the vertical angle is $+3° 0'$. Enter Fig. 126 on the lower scale at 302 and run vertically to the slanting line marked 3°. The difference in elevation is read on the vertical scale by interpolation as 15.9 feet. To this should be added about 0.1 foot for each 5 degrees of vertical angle to allow for the term $(f+c) \sin a$ in the stadia formula, as the diagram gives only the value of $100\ S \sin a \cos a$ (see Theory of Stadia, Page 117, "Plane Surveying", Part II).

It is noted also that the intersection of the 302 line and the 3° line comes very close to the line marked "one". This means that 1 must be subtracted from 302 to get the first term in the horizontal distance formula $100\ S \cos^2 a$. To this must be added the value of $(f+c) \cos a$, which can ordinarily be taken as 1, or unity, to get the horizontal distance which in the above case is $302-1+1$, or 302 feet. Differences in elevation usually are carried to tenths, but horizontal distances are carried only to feet. For distances less than 100 feet use diagram between 100 and 1000, dividing result by 10.

Example. Reduce the stadia readings given as examples in the explanation of the stadia tables, page 123, "Plane Surveying", Part II, by means of the diagram, and note how closely they check.

TOPOGRAPHY BY PLANE TABLE

Function. The plane table is used mainly in government and state surveys and is often not found in the ordinary engineering office. It consists essentially of a drawing board mounted on a tripod and a ruler provided with sights, etc., called the *alidade.*

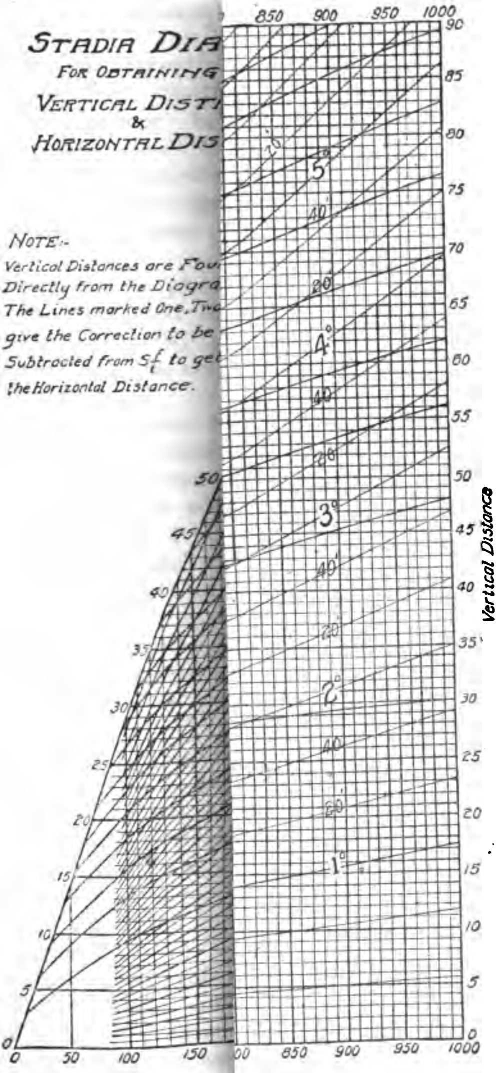
STADIA DIA

FOR OBTAINING

VERTICAL DIST
&
HORIZONTAL DIS

NOTE:-

Vertical Distances are Fou
Directly from the Diagra
The Lines marked One, Two
give the Correction to be
Subtracted from $S\frac{f}{i}$ to ge
the Horizontal Distance.



All reductions of observations are made and the map is drawn directly in the field. Stations in the field are located on the map by traversing or by graphic triangulation. The chief advantage of the plane table is that the map is drawn in the field and better results can be obtained with fewer shots than in the transit-and-stadia method. It is used mainly for topographic surveying over large areas where close detail is not required.

Construction. The plane-table board is usually 24 by 30 inches and is constructed in sections to prevent warping; it is attached to the tripod by a three-screw leveling base arranged to permit the board to be turned in azimuth and to be clamped in any position.

The alidade is designed to at once sketch to scale, in the field, the lengths and relative directions of all lines and the positions of objects to be included in the survey. For drawing straight lines, a steel ruler is provided on which is mounted, at each end, a pair of open sights like those of the compass; or a telescope, fitted with stadia wires, a vertical arc, and a level bubble, is mounted at the center of the ruler. The line of sight should always be parallel to the edge of the ruler:

For leveling the instrument, two level bubbles at right angles to each other are mounted upon the alidade, and either an attached or a detached compass is provided for determining the bearing of lines.

For attaching the paper to the board, various devices are used. One consists of a roller at each end of the table, on one of which the paper is wound as it is unrolled from the other, the edges of the paper being held close to the board by spring clips. The arrangement permits the paper to be used in a continuous roll. Separate sheets of proper size can also be used, attached to the board and held firmly in place by the spring clips provided with the instrument.

Under the most favorable conditions, the plane table is a very awkward instrument and difficult to carry, but it is admirably adapted to filling-in the details of a topographic survey; for this purpose it is the standard instrument of the U.S. Geodetic Survey and is also largely used by the U.S. Geological Survey. It cannot be used on damp or very windy days and, consequently, is not so generally utilized as the transit and stadia.

In Fig. 127 is shown one form of construction of the plane table, with a Johnson leveling head.

Adjustments. (1) *To determine whether the edge of the ruler is straight.* Place the ruler on a smooth surface, draw a line along its edge, and also draw lines at its ends. Reverse the ruler on these lines, and draw another line along its edge. If the two lines coincide, the ruler is straight.

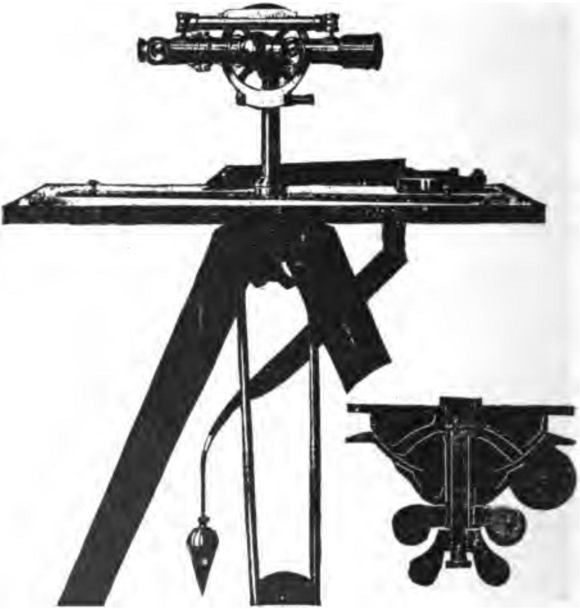


Fig. 127. Typical Plane Table
Courtesy of W. and L. E. Gurley, Troy, New York

(2) *To make the plane of the table horizontal when the bubbles are at the middle of the tubes.* Assuming the table to be plane, set the alidade in the middle of the table, level by means of the leveling head or leveling screws, draw lines along the edge and the

ends of the ruler, and reverse the alidade on these lines. If the bubbles remain in the middle of the tubes, they are in adjustment. If they do not, correct one-half the error by means of the leveling head or screws and the remainder by means of the capstan-headed screws of the level tubes. Repeat the operation until the bubbles remain in the middle of the tubes in both positions of the alidade. This is simply the principle of reversion, Fig. 29, "Plane Surveying", Part I.

(3) *To make the line of sight perpendicular to the horizontal axis of the telescope.* Level the table and point the telescope toward some small and well-defined object at a distance. Remove the screws which confine the horizontal axis of the telescope in its bearings, reverse the telescope in its bearings, that is, change the axis end for end, being careful not to disturb the position of the alidade upon the table, and again sight on the same object. If the intersection of the cross hairs bisects the object, the adjustment is complete. If not, correct one-half the error by means of the capstan-headed screws attached to the cross-hair ring. Sight on the object again and repeat the operation until the line of sight will bisect the object in both positions of the telescope.

(4) *To make the line of sight parallel to the axis of the bubble tube.* Carry out the adjustment by the peg method as described for the transit, "Plane Surveying", Part II.

(5) *To make the horizontal axis of the telescope parallel to the plane of the table.* Level the table and point the telescope to a well-defined mark at the top of some tall object. Turn the telescope on its horizontal axis, point it to a small mark at the base of the same object. Draw lines on the table at the edge and at the ends of the ruler. Reverse the instrument on these lines, point the telescope to the upper mark, and turn it on its horizontal axis. If the line of sight again covers the lower point, the adjustment is complete. If it does not, adjust the horizontal axis until the line of sight when plunged from the high point comes midway between the two lower points.

(6) *To make the vertical arc or circle read zero when the line of sight is horizontal.* Correct this by means of the screws attached to the vernier plate in the same manner as for a transit.

Use in Traversing. The plane-table method of traversing is practically the same as the method of surveying a series of lines with the transit. It is the best method of working as it provides a complete check on the survey.

Let *ABCDE*, Fig. 128, be the series of lines to be surveyed by traversing. Note that in this illustration and the ones following, large letters represent points in the field and small letters represent the plotted points on the map. Set up the table at *B*, the second angle of the line, so that the point *b* on the paper will be directly over the point *B* on the ground. Stick a needle at the point *b* and place the edge of the alidade against it. Swing the alidade around until the line of sight covers the point *A*. Measure *BA* by stadia (see page 117, "Plane Surveying", Part II), and lay it off to the proper scale as *ba*. Now turn the alidade around the point *b* and sight to and measure the distance *BC* by stadia and plot it to scale as *bc*. Remove the instrument to *c* with the point *c* on the paper directly over *C* on the ground. This is difficult to accomplish with the plane table, but if the plat is drawn to a large scale, it must be done. The board must now be "oriented", that is, placed in a parallel position to its position when at *B*. This is done by putting a needle at *c*, setting the alidade on the line *cb*, and turning the board until the line of sight cuts *B*. Clamp the board securely in this position. Check the readings to *B*. Swing the alidade around *c* until the line of sight covers *D*, measure *CD*, and plot *cd*. Remove to *D* and proceed as before, and so on through the traverse.

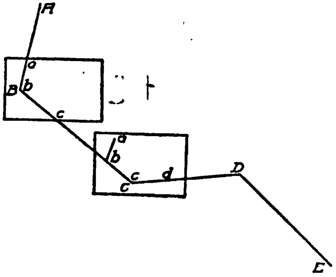


Fig. 128. Diagram Showing Use of Plane Table in Traversing

If the problem represents the survey of a closed field, the engineer will have the advantage of the closure of the survey as a check on the work.

Intersection. Intersection is simply graphic triangulation. Set up the instrument at any convenient point, as at *A* in Fig. 129, sight to all the desired points, as *D*, *E*, *F*, etc., which are visible, and draw indefinite lines in their directions. Measure any line, as *AB*, *B* being one of the points sighted to, and plot the length of this line on the paper to any convenient scale. Move the instrument to *B* so that *b* on the paper will be directly over *B* on the ground, and so that *ba* on the paper will be in the direction of *BA* on the ground. Stick a needle at the point *b* and swing the alidade around it, sighting to all the former points in succession, and draw lines in their direction. The intersection of these two sets of lines to the several points will determine the position of the points. In choosing the base line *AB*, care must be exercised to avoid very acute or obtuse angles, 30° and 150° being the extreme limits.

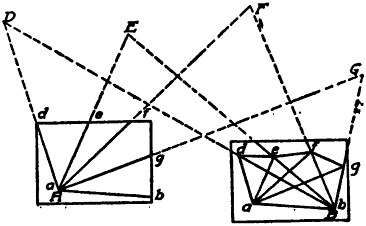
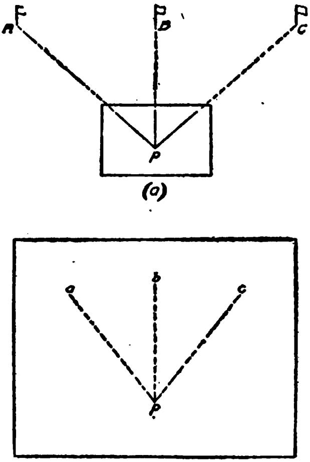


Fig. 129. Diagram Showing Use of Plane Table in Intersection

Three-Point Problem. When the location of three points on the paper or map is known, the plane table may be moved to any place from which these three points can be seen, and the proper location of this point may be found on the map. Two methods are available, as follows:

Tracing-Cloth Method. A piece of tracing paper or cloth is stretched over the board which is then oriented so as to be, as near as can be judged, in its proper position. A point *p*, Fig. 130, is then selected on the board to represent, as near as can be estimated, the location of the point of set up on the paper. From



(b)

Fig. 130. Sketch Showing Solution of Three-Point Problem by Tracing-Cloth Method

this point lines are drawn on the tracing cloth to the three known signals or points *A*, *B*, and *C*, Fig. 130(*a*). The tracing cloth is then loosened and moved around until the three lines pass through the three plotted points *a*, *b*, and *c* on the map, Fig. 130(*b*). The point on the map under *p* is then the correct point.

The board should then be correctly oriented or set in azimuth by placing the rule on *a* and *p* and turning the board so that the telescope sights at *A*. As a check, pointings from *p* to *B* and *C* should pass through the plotted points *b* and *c*.

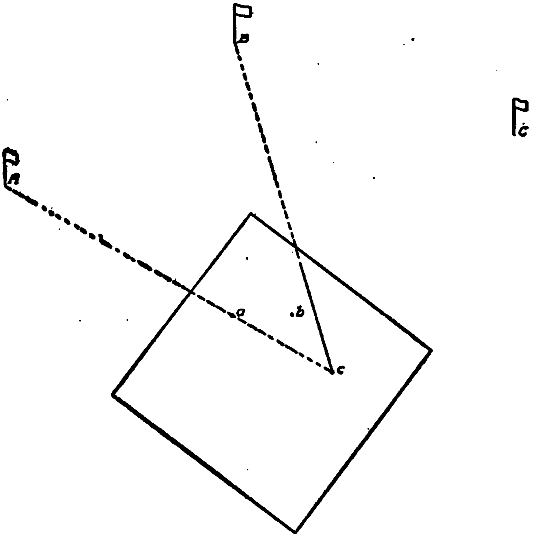


Fig. 131. Plane-Table Diagram Showing First Step in Bessel's Solution

Bessel's Solution. Bessel's solution is a rather difficult method but will be understood by carefully studying out the three following steps:

(1) Set up and level the board with the edge of the alidade through *c* and *a*, Fig. 131, turn the board until the line of sight strikes *A*, then clamp. With the edge through *c*, sight *B* and draw an indefinite line.

(2) Unclamp. With the straightedge through *a* and *c*, Fig. 132, turn the board until the line of sight strikes *C*, then clamp. With the edge through *a*, sight *B* and draw a line intersecting the line drawn from *c* in the point *e*.

(3) Unclamp. With the straightedge through *e* and *b*, Fig. 133, turn the board until the line of sight strikes *B*, then clamp. With the edge through *a*, sight *A* and reset on *eb* produced. This gives the desired point *p*. By sighting, check *C*. With the edge through *c*, the line *Cc* produced should pass through *p*. The board is now correctly oriented, and the point *p* is located.

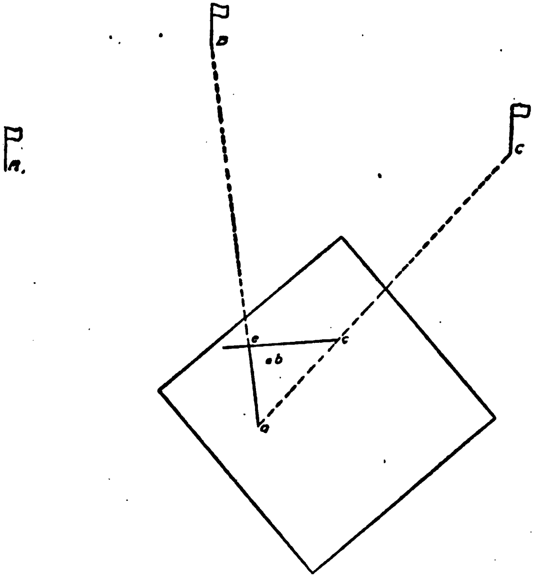


Fig. 132. Plane-Table Diagram Showing Second Step in Bessel's Solution

A disadvantage of this solution is that the point *e* may fall too close to *b* to give a good line. In such a case, it is well to use the tracing-cloth solution.

Radiation. Having located the point of set up on the paper, side shots may be taken to locate topography. This differs in no way from taking side shots with a transit by stadia, except that azimuths are not read. The directions are plotted directly on the paper.

HYDROGRAPHIC SURVEYING

Operations in Hydrographic Surveying. Hydrographic work has to do with surveys of bodies of water. In many cases, a topographic survey involves some hydrographic work, as, for example, when a survey of a lake is made to determine how much water could be stored up by a dam at the outlet. Here it would be necessary to take contours all around the shore, so that the volume could be computed by averaging the areas inside two adjacent contours and multiplying by the contour interval and so that the new shore line could be determined. One of the principal operations in hydrographic work is the taking of soundings, either to

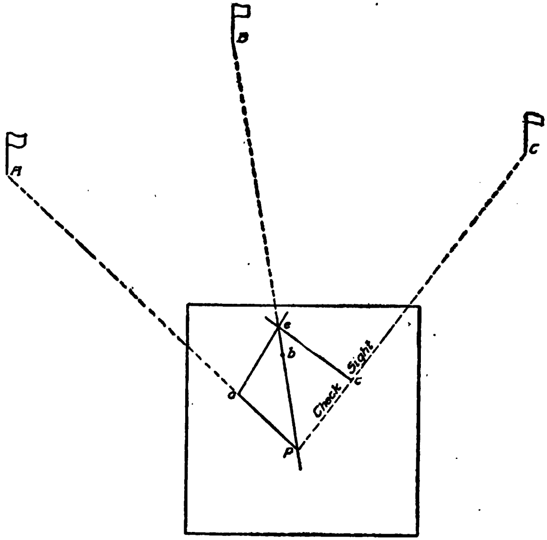


Fig. 133. Plane-Table Diagram Showing Third Step in Bessel's Solution

determine the depth of water for navigation, or for computing the amount of material to be dredged, etc. The latter computation is made in much the same manner as that described for borrow pits, Fig. 114, "Plane Surveying", Part III. The measurement of currents and stream flow also is frequently included under hydrographic surveying, but it involves a knowledge of the subject of hydraulics and is usually treated under that heading.

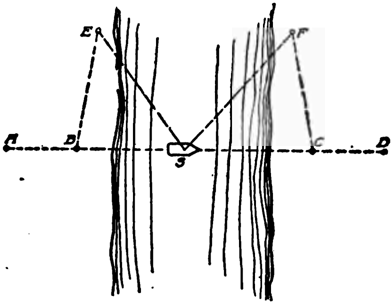


Fig. 134. Diagram Showing Intersection Method of Making Soundings

Soundings. Deep-sea soundings are made with special and elaborate apparatus. Ordinary soundings in moderately deep water are made with a lead weight attached to a graduated line or chain. Heavy cord that has been well wet several times and wrapped around a post or tree to dry and stretch will make a good line. For soundings in water less than 12 feet deep a rod is used.

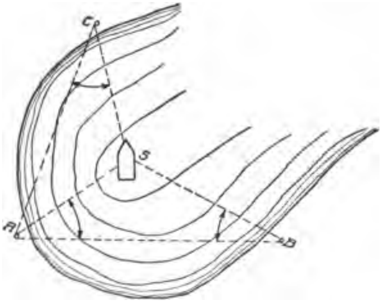


Fig. 135. Diagram Showing Angle Method of Making Soundings

Several methods are available for locating soundings. All involve first the location of two or more properly placed points on shore, and such location is made usually by triangulation. Also, it is usually necessary to establish a series of parallel "range lines" at the required interval apart, along which the sounding boat is moved, the soundings being taken at certain intervals of time. This insures that the soundings will be uniformly distributed over the area.

Intersection Method. A line *AD*, Fig. 134, is established by poles set at *A*, *B*, *C*, and *D*. The sounding boat proceeds along this line, keeping in range by means of the poles; its position relative to station *E*, for example, when a sounding is taken, is determined by measuring the angle *BES*. As a check, the angle *SFC* from another point *F* should also be measured. The relative positions of *B*, *E*, *F*, and *C* being known, they may be plotted on the map, these angles laid off, and the point located.

Angle Method. The angle method involves reading the angles on the boat from three known points on the shore, the third point being used as a check. Thus the angles, Fig. 135, at *A* and *B* locate *S*, and that at *C* checks the location.

Three-Point Method. The three-point method is the same as the three-point problem in plane-table work. When the sounding is taken, the two angles at the boat *S*, Fig. 136, between three known stations on the shore are read. As a check, the third angle also should be read. For this purpose a sextant, an angle-measuring instrument that can be held in the hand and is used by navigators, is employed. Having the three points plotted on the map, *S* may be found by the tracing-cloth method or by using a three-arm protractor. The distance from *S* to the signals may also be computed.

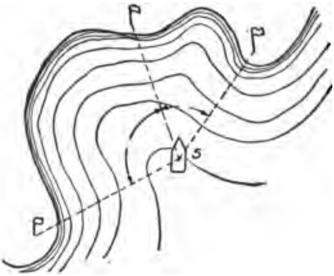


Fig. 136. Diagram Showing Three-Point Method of Making Soundings

The three-point method is used for surveys on large bodies of water, while the intersection and angle methods are employed for small lake and river work.

TRIANGULATION

Fundamental Operation. The fundamental operation in triangulation consists in measuring all the angles and one side of a triangle, from which measurements of the other two sides may be computed. This method is used to determine distances that cannot be measured directly, as, for example, the distance across a river, Fig. 137, and a series of triangulation stations, determined in the same way, is used as a basis for extended topographic and other surveys.

In Fig. 137 the distance AB is required, say, for a bridge which is to span the stream. A stretch of level ground is found from B to C, so the length BC is measured. This forms the known side, or base line. The angles at A, B, and C are then measured, from which data the distance AB is readily computed by the sine formula

$$AB=\frac{BC\times\sin ACB}{\sin BAC}$$

As a check, another base line AD could be measured, and AB could be computed from the triangle ABD. If it is not convenient to measure the distance AD, a check may be obtained by measuring the angles BDC and ACD. AB is then obtained from the triangle ABC as before, and also by first solving BDC for BD, and then ABD for AB with BD known.

Other cases that may be solved in a similar manner will suggest themselves to the student.

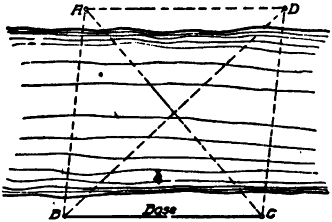


Fig. 137 Diagram Showing Determination of Distance across River by Triangulation

In the case of topographic work over a large area, if the work is done by traversing by transit and stadia, the accuracy cannot be expected to be much better than 1 in 1500. If the area is, say, 1 mile square, this would mean that points on the extreme edges might be off by 3 or 4 feet, which would show on the map if it were plotted to a large scale. In this case the traverse could be chained, which would decrease the error to, say, 1 in 4000 or 5000. If the error were still larger, it would be advisable to locate accurately certain stations like C, D, E, and F, Fig. 138(*a*). The stadia traverse would then be run between these stations, and the traverse error would not be carried on. Thus the instrument might be placed at E, the azimuth of EF determined, and an open traverse run from E to F. The azimuths could be checked by setting up at *F* and observing the back azimuth to *E*, and the traverse itself could be checked by computing *FE* as its closing side and comparing this with the value of *FE* from triangulation. If the error were satisfactory, the traverse would be balanced to the triangulated value of *EF*, and a new traverse would be started at *E* to run to *D*, etc.

In Fig. 138(*a*) is shown a scheme of triangulation for locating these stations. *AB* is the base line. The triangles *ABC* and *ABD* would be computed first, and from these the diagonal *CD*, which serves as a base for the triangle *CDE*. Thus *CD* may be found from the triangle *CBD*, in which *CB* and *BD* and also the angle *CBD* are known. This solution may be most easily made by the method shown in Fig. 138(*b*). Drop a perpendicular *DX* from *D* on *CB* prolonged. Solve the right triangle *BDX* for *BX* and *DX*. Then, in the right triangle *CDX*, we know *DX* and *CX*—which equals *CB*+*BX*—and can solve for *CD*.

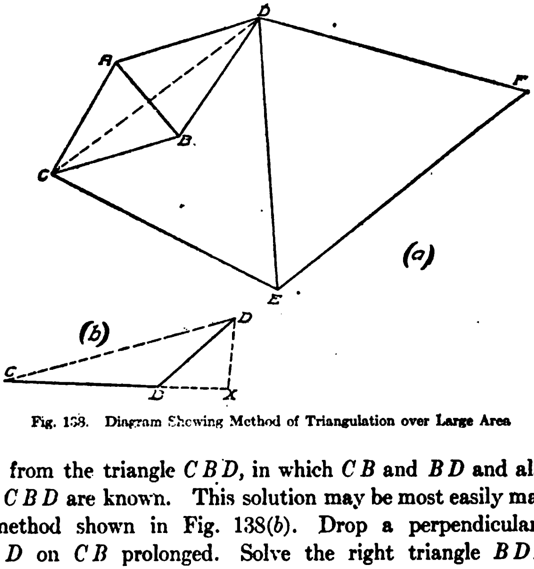


Fig. 138. Diagram Showing Method of Triangulation over Large Area

Selection of Stations. In selecting stations, we must take care to first locate the base line on clear uniform ground where it can be easily and accurately measured. The remaining stations are then located so as to get a good expansion from the base and to be convenient for the purposes of the survey. The best triangles have angles of 60 degrees. Long slender triangles are to be avoided, and no angle in a triangle should be less than 30 degrees nor greater than 120 degrees.

Base Line. It is clear from what has been stated that the scale of the entire survey depends on the accuracy with which the base line is measured. The most accurate methods, therefore, are used in its measurement, as it is often of small length compared to the distance between some of the triangulation stations.

The most accurate method is to use a tape supported at intervals by nails driven into small posts, as shown at *C*, *D*, and *E*, Fig. 139. At the end of each tape length a heavy post or table is driven, and on top of these surfaces are placed small copper strips on which a scratch may be made opposite the end of the tape. The zero of the tape is held at *B*, exactly opposite one of these marks, by means of an iron range pole held under the arm and to which the tape is tied. The proper pull is put on the tape, using a spring balance at *A*, and a scratch is made on the copper strip at its end on the table. The tape is then moved along and its zero end held at *A*, etc. The tables may be set at any convenient height, but the points of support should be on the line between the table tops.

The reduction of observations consists in allowing for temperature, pull, and sag, and finally in reducing to horizontal distances by using the difference in elevation of the tables obtained by leveling. When a tape is used for much work of this kind, it is preferable to have it standardized by the U.S. Bureau of Standards, Washington, D.C., the tape being supported at the same points and under the same pull used in the field. This eliminates the sag and pull corrections. The tem-

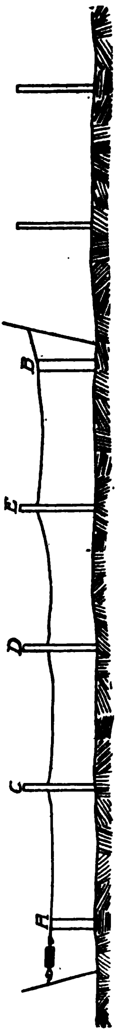


Fig. 139. Method of Supporting Tape in Measuring Base Line

perature is very important in accurate work, and the taping should be done at night or on cloudy days. Tapes of invar steel can also be used, but they are too expensive to justify their use unless a great deal of base work is done.

When all the results have been reduced for several independent measurements of the base, it will, of course, be found that they do not agree. The best value to use is the mean, and some idea of the accuracy of the mean may be had from the following formula, which gives what is called the *probable error of the mean* of a series of observations and is

$$E_m = \pm 0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}$$

in which Σv^2 is the sum of the squares of the differences between each measurement and the mean, and n is the number of measurements. This formula allows only for accidental errors. All constant errors must be eliminated in making the measurements. The probable error of a single measurement would be

$$E_s = +E_m\sqrt{n}$$

By the method of taping described above, an accuracy of 1 in 100,000 or better can readily be obtained. With an invar tape, work has been done with a precision of 1 in 1,000,000 and better.

Examples. 1. The following measurements of a base line were made: 4198.62; 4198.63; 4198.57; 4198.61; and 4198.58. Find the mean and its probable error. Ans. 4198.602 ± 0.008, or about 1 in 500,000

2. Find the probable error of a single observation made under the same conditions as those in the above example. Ans. 0.018, or 1 in 240,000

Angles. The angles at each station should be read by repetition with a transit reading to 30″ or better. Ordinarily, 4 or 6 repetitions are all that are used. Each angle is read and also its explement—the amount necessary to complete the round angle. Thus the angle AOB, Fig. 140, is read, then the angle BOA around through C, D, E, and F is read. With careful work, these should check to within 10″ of 360°. The error is divided equally between them. Next the angles BOC and COB are measured and adjusted, etc. Finally the sum of all the angles, AOB, BOC, etc., should equal 360° to within 10 or 15″ The error, if satisfactory, is distributed equally among them.

In addition to the station adjustment of the angles as mentioned above, there is generally found an error, when the angles in any triangle are added up, by which they fail to equal 180°. This error will seldom be more than the station error and is corrected by the triangle adjustment which puts one-third of the error in each angle. These are the final values used in the computations.

It will be noted in the above that the triangle adjustment probably spoils the station adjustment, and in other cases—for example, if the angles of the diagonal *CD*, Fig. 138, were read—it is necessary that the angles should add up properly in more than one triangle. The observed values may be adjusted so they will satisfy all these conditions by the "Method of Least Squares", *Special Publication No. 28* of U.S. Coast and Geodetic Survey, but such a computation is very laborious and its use is warranted only in work of great precision, such as in geodetic surveying.

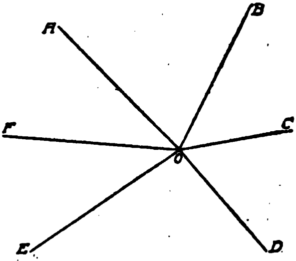


Fig. 140. Diagram Showing Angles in Each Station

In the scheme of triangulation shown in Fig. 138, the only check on the accuracy of the work is the closure of the angles. One of the other sides of the figure may be measured as a check base, if it is desired.

PLOTTING AND MAPPING

DRAWING INSTRUCTIONS

Since all students who study this course have already taken that on Mechanical Drafting, only such instruction will be given here as is required in addition to instructions in the text on Mechanical Drafting.

Pens and Pencils. In addition to the usual set of instruments, a contour pen, Fig. 141(a), is necessary for topographic work. This is a swivel-nib pen. When in use, it should be held nearly vertical in the hand, with the little finger resting on the paper and taking the weight of the hand. Considerable practice is necessary in order to follow the curves of a contour and make a smooth line.

In order that roads may be drawn to follow the proper curves and the sides be perfectly parallel, a road pen, Fig. 141(*b*), also is desirable. This pen, like the contour pen, is fastened to the handle so as to be free to turn and is used in the same way.

As a large amount of map work is free-hand, a liberal supply of pens suitable for work of this kind is needed. Students should not use the extremely fine crow-quill pens. Gillott pens, No. 303 and No. 404, are the best, the former for tracing cloth and the latter for paper. They make a solid definite clear line that will blue print satisfactorily from a tracing.

A 3H pencil is usually hard enough for most work of this kind. The ordinary rubber eraser is indispensable, and for cleaning drawings on paper, a soft sponge eraser and art gum are very useful; a small can of benzine or carbona can be used to remove pencil marks and dirt from tracings if waterproof ink has been used.

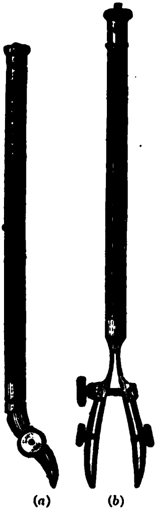

(*a*) (*b*)

Fig. 141. (*a*) Swivel-Nib Pen; (*b*) Road Pen

Paper and Ink. It was formerly the practice to make maps on paper, but at present most work of this kind is done on tracing cloth so that reproductions may be obtained by blue printing. It is always desirable to use the best cloth as the cheaper grades shrink or swell with the weather changes. A small amount of Fuller's earth, which may be purchased at any drug store, dusted over the dull side of the cloth, which should always be used, removes the surface gloss and makes the ink take more readily.

When paper is used, it is desirable to use a paper with a fairly smooth and not a hairy surface. An eggshell paper is probably best. For maps that are to be handled much, the paper should be *mounted,* that is, backed with linen.

Waterproof ink should always be used. India, or black, vermilion, blue, and brick red; the brick red is required for contours, etc. Ink should always be kept tightly corked.

Scales. Usually *decimal* scales are used for plotting topographical work, since the maps are frequently drawn on a scale of 100 feet, 200 feet, 400 feet, or 500 feet per inch. Therefore a triangular decimal scale with its six scales graduated to 10, 20, 30, 40, 50, and 60 parts per inch is, in general, the most useful for that purpose. Even if dimensions are scaled off in hundredths of an inch, the simplest plan is to use the "50" scale, in which each division means $\frac{1}{50}$, or 0.02 inch. When maps are drawn to some special ratio, as, for example, 1 : 125,000 or 1 : 62,500, it will facilitate the work and it will be justifiable to construct special scales which will permit the distance in feet, as measured in the field, to be scaled off directly on the map. For example, the ratio 1 : 125,000 means 1 foot on the map equals 125,000 feet on the ground, or 1 inch on the map equals 10,417 feet on the ground. Where the work to be done is very extensive and it must all be done at the same scale, there is considerable advantage in using a *flat* scale rather than a *triangular* scale, since, under such conditions, the five other scales of the triangular scale are useless and a blunder is frequently caused by inadvertently using the wrong scale. Metal scales have the advantage of extreme durability but the disadvantage of rusting and tending to soil the paper. The boxwood scales having a natural wood color are the most common, but they grow darker with age and with handling, and the graduations become so dim that it is difficult to use them. Probably the best scales are those made of boxwood but have the beveled edges covered with a material resembling ivory, which permanently remains white. The scale to which a map is drawn should always be clearly shown on the map, usually in the title. It is also best to show it graphically by a device such as is shown in Fig. 142.

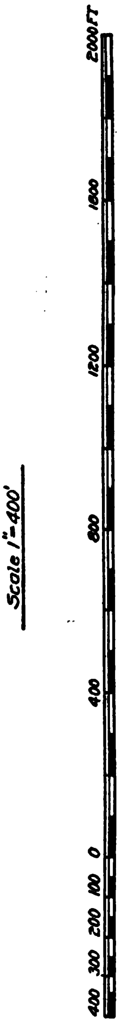


Fig. 142. Typical Sketch for Scale Placed on Topographic Maps

Protractors. The very small metal protractors frequently included in a set of drawing instruments are almost useless for mapping work. Celluloid, or xylonite, protractors which are at least 6 inches in diameter are often convenient; but for accurate plotting large paper protractors 14 inches in diameter printed on Bristol board which can be bought for 40 cents are much more useful. Protractors which may be set by a tangent screw and vernier to a reading of 5′ can be obtained but are very expensive and represent an attempt at accuracy that can seldom be attained in drafting work.

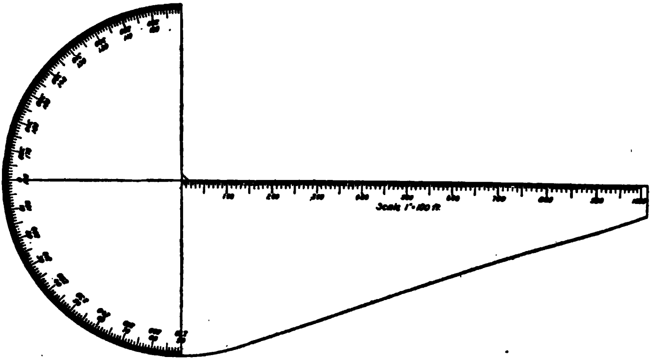


Fig. 143. Modified Sketch of Molitor Protractor

For plotting topography shots taken by azimuth and stadia, the best protractor is Molitor's a slight modification of which is shown in Fig. 143. A needle fixes this protractor to the drawing board at its center, about which it can be turned. A zero azimuth line is drawn through the station from which side shots are to be plotted and the protractor is centered over this station. When the protractor is swung until the meridian line is under the recorded azimuth on the protractor, the scale arm then lies in the recorded azimuth, and the distance out may be scaled and the point plotted. It will be plain from the arc numerals that azimuths from 0 to 90° and from 270° to 360° are set at the south end of the zero azimuth line, while those from 90° to 270° are read from the north end. One of these protractors may easily be made by pasting an arm on a small paper protractor and graduating this arm to suit the scale used.

The three-arm protractor, Fig. 144, is of use in solving the three-point problem instead of using tracing cloth. Few offices, however, have enough work of this kind to warrant the purchase of one of these protractors.

Lettering. Many a map which is otherwise good is spoiled by poor lettering. "Mechanical Drawing". Part III, contains a treatise on lettering and, therefore, such instruction will not be repeated here. In general, it should be said that the style of lettering adopted should be simple and as plain as possible. No fancy lettering should be used except, perhaps, in the title, and even there some forms of fancy lettering are not only in bad taste but are a useless waste of time and skill. The illustrations of government maps given herewith should be studied particularly with reference to the style of lettering used for different topographical features. Where much lettering is necessary—for example, to indicate the location of numerous towns on a small scale map—the simplest form of line lettering is preferable. When possible, all lettering should run horizontal with the base of the map,

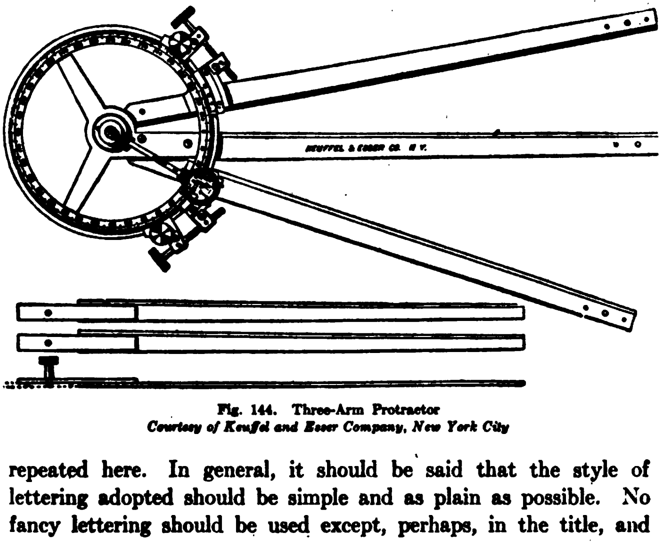


Fig. 144. Three-Arm Protractor
Courtesy of Keuffel and Esser Company, New York City

except the lettering to indicate the names of streams, railroads, etc., which should follow their course. The spacing between letters used to indicate the names of streams and railroads should likewise be strung out to cover a considerable part of the length of the stream or railroad unless that spacing would be so great as to make it difficult to follow the continuity of the letters.

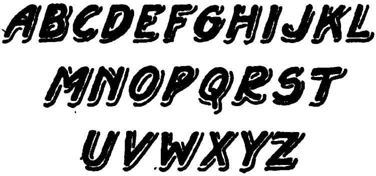


Fig. 145. Typical Free-Hand Lettering for Land Maps

Heavy free-hand lettering with shade lines, such as shown in Fig. 145, is very easy to make well and can be used for the main portion of titles on farm and real-estate maps. It is probably too fancy for many other maps, and the student should practice on the plain block lettering and, for very important and formal work, on the Roman letters which are used on all government work which, while very difficult to do well, are the best form.

Border Line. A simple border line which consists of a single heavy line of uniform width or of two lines, as shown in Fig. 146, will add to the neatness of a map, and it should usually be made. When it is specially desired to attract the eye of nontechnical people, as in the case of real-estate maps, which are made to look as attractive as possible to catch the eye of a possible purchaser, a very elaborate border line may be justifiable, but otherwise it must be considered a waste of time and skill and even an evidence of bad taste.

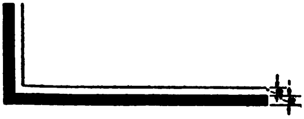


Fig. 146. Simple Border Line for Maps

Pointers. Pointers are almost always required on maps to show the direction of the meridian. Very fancy designs are sometimes used but should be avoided. The simple form shown in Fig. 147(*a*) may be used for land and other maps that are not heavily covered with details. For topographic maps, where a great deal of detail is shown, the more elaborate pointer, Fig. 147(*b*), may be used. Note the proportions of the parts of these pointers.

Titles. The map should always contain a complete title and, in addition, the name and address of the surveyor, the scale, date, and pointer, and notes as to how the survey was made. On topographic maps it is well also to give a legend or samples of the conventional signs used on the map, similar to those shown in Plates II, III, etc. It is also desirable to state on topographic maps the datum on which the contour elevations are based and to show the location and the elevation of several bench marks.

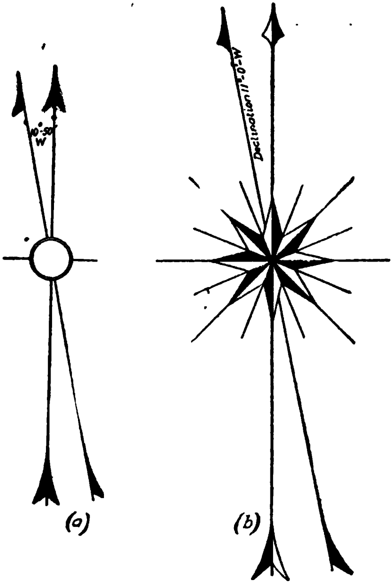


Fig. 147. Typical Pointer Designs

PLOTTING

Traverse Stations. Traverse stations should preferably be plotted from co-ordinates. No explanation of this method is required; but it should be noted that it is usually best to use a T-square and right triangle rather than to rely on two edges of the drawing board being exactly at right angles. If traverses are plotted from angles proceeding around the traverse, it will always be found that there is an error of closure in the plotting, owing principally to errors in laying off the angles. If this method is to be used, either a very accurate protractor must be employed or the angles may be laid off by the tangent method illustrated in Fig. 148, using a table of natural tangents and a decimal scale of, say, 20 to the inch. A 5-inch length of this scale is divided into 100 parts and by interpolation it may be read to 1000 parts. Lay off the 5 inches on a line and erect a perpendicular to that line at the 5-inch point. Suppose that the angle to be laid off is 16° 45′; the natural tangent taken from a table is 0.301. Therefore, using the 20 scale, scale off on the perpendicular 30.1 divisions. A line to the starting point will make the required angle of 16° 45′, with the given line. Since the length of the perpendicular would be unreasonably long with angles which approach 90°, the preferable method would be to erect a perpendicular to the given line at the point from which the angle was to be made; lay off 5 inches on that perpendicular, erect a line perpendicular to it, or parallel to the original line, and determine the tangent of 90° minus the given angle. Of course, if still greater accuracy is desired, a longer base line than 5 inches can be used.

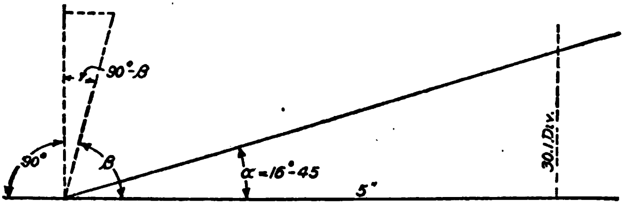


Fig. 148. Plotting Angles from Tangent

Other details of the plotting of farm surveys, which are applicable to other forms of plotting, are given subsequently.

Farm Surveys. Farm surveys usually consist only of boundaries and are best plotted by co-ordinates as described above. The data shown is illustrated in Fig. 101, "Plane Surveying", Part III.

Stadia Shots. The use of the Molitor protractor for plotting stadia shots has already been noted. The plotting of contours from stadia work, however, requires some explanation. It will be remembered that contours are determined by stadia shots to points so located that the ground has a uniform slope between them; hence all points on a line between contour points will lie approximately in the surface of the ground. On this assumption, the location of any contour which comes between two located points is a mere matter of dividing the horizontal distance between the points into parts which are proportional to the differences of elevation of the given points and the contour. For example, in Fig. 149, the elevation of one point is 162.0 and of another 167.8. The 165-foot contour, evidently, will lie between them and at $\frac{165.0-162.0}{167.8-162.0}$, or $\frac{3.0}{5.8}$ of the horizontal distance between them from the lower point; by similar triangles it cuts the slope in the same ratio. This is the theoretically exact solution but, since it depends on an approximation, the experienced draftsman can make a rapid mental solution of the proportion which will be as accurate as the data justify. If there are two or more intermediate contours, the same general method may be applied for each, but the simplest method will be first to locate the contours immediately adjacent to each point and then to interpolate any intermediate contours. After determining where each contour will cross the line joining each pair of surveyed points, the contours may be drawn in by connecting the corresponding intersections. Practical modifications of this theoretical method will be made by the topographer or the draftsman to allow for minute variations in the contours, which may be sketched in with sufficient accuracy, since the area between determined points will always be small.

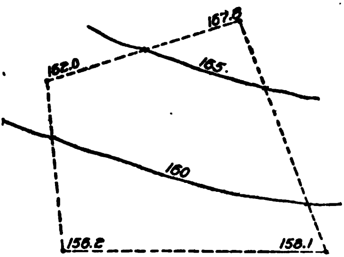


Fig. 140. Plotting Contours

Profiles. Profiles are almost invariably plotted on profile paper. There are three general methods of ruling the lines on profile paper, and the paper adopted will depend somewhat upon the use to be made of it. Vertical lines are ruled $\frac{1}{4}$ inch apart and sometimes $\frac{1}{5}$ inch apart, Fig. 150. Horizontal lines are ruled 20, 25, or 30 to the inch. The paper comes either in sheets about 42 inches long and 15 inches high or in rolls 10 or 20 inches wide and 50 yards long. Sometimes the paper is mounted on muslin, and sometimes the rulings are made on tracing paper or on tracing cloth. The levels are always referred to some datum plane which is lower than any elevation to be recorded. Each fifth horizontal line is ruled somewhat heavier than the others, and one of these lines should be selected as the line indicating some even multiple of 5 feet above datum plane. Some little ingenuity often is necessary to choose the proper line so that the profile will run neither too high nor too low. The scales, both horizontal and vertical, which are to be used must be selected with reference to the character of the work to be shown. The horizontal scale will usually vary from 100 to 400 feet to the inch. On a scale of 400 feet to

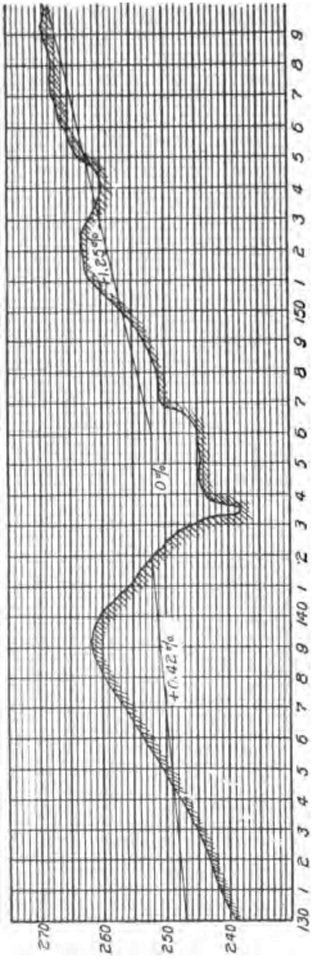


Fig. 150. Sketch Showing Method of Laying Off Profiles

the inch, each vertical line corresponds to 100 feet of horizontal distance. Usually each horizontal line—spaces $\frac{1}{20}$ inch apart—means 1 foot of elevation or, in other words, the vertical scale is 20 feet to the inch. Since the engraved lines usually cover a vertical space of 15 inches, this scale will allow a range of profile of 300 feet. If 20 feet per inch is chosen as the vertical scale and the total vertical range of the profile does not exceed 300 feet, then the whole profile may be placed without a break on such a sheet, but the choice of lines to represent any given elevation must be made accordingly. When a larger vertical scale is chosen or the range of elevation is greater, as it is likely to be for a continuous railroad profile, a break may be made by raising or lowering the profile an even number of feet—for example, by making a change at some vertical line, by which the elevation indicated by a given horizontal line is abruptly changed, say, 100 or 200 feet.

Mark on the profile sheet the elevation corresponding to the horizontal lines and also the distances corresponding to the vertical lines (it is the usual practice to mark each 10th station and each 10-foot elevation on the profile map), then having recorded in the notebook the elevations above the datum plane of every desired point in the profile, plot each point at its proper position according to the horizontal scale and likewise according to the vertical scale. A line drawn through these several points will give the profile of the surface. If it is desired to establish a given grade line on that profile, the location in distance and elevation of the governing points of that grade line should then be plotted and the grade line drawn through them. Usually this grade line will lie alternately above and below the surface line, and we thus have a graphical representation of the amount of cut or fill required at any point.

PROJECTION

For small areas, such as farms and parks where the area is but a few square miles, it is sufficiently accurate to assume that the surface of the earth is plane or, in other words, that all plumb lines held at all points of the map would be precisely parallel. But when the areas amount to hundreds of square miles and, particularly, when they cover whole continents, the curvature of the earth cannot be neglected either in the computations or in plotting. A single-curved surface, such as that of a cone or cylinder, can be developed and it will lie flat, that is, the surface could be rolled out on a plane surface without changing any proportions. But a surface of double curvature, such as a sphere, is not developable, and there then arises the problem of representing on a plane surface the surface of a sphere with as little distortion as possible. There are many methods of accomplishing this, some of which are very approximate. The problem of projection never comes up in ordinary work, as it is only for very large areas that curvature must be taken into account. The student is therefore referred to textbooks on "Geodetic Surveying" for data on this subject and also to "Tables for a Polyconic Projection of Maps", *Special Publication No. 5*, U.S. Coast and Geodetic Survey.

SPECIAL INSTRUMENTS

Pantograph. The pantograph is a very useful and essential instrument when drawings are to be redrawn on a different scale.

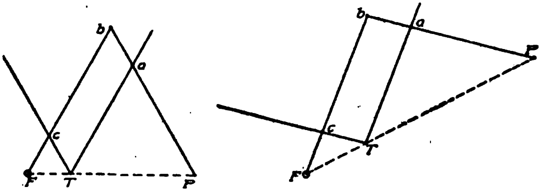


Fig. 151. Sketches Showing Principle of Pantograph

It frequently happens that field drawings, especially when made with the plane table, are drawn to a larger scale than is used for the final engraved maps. The principle of the pantograph is as shown in Fig. 151, in which F represents the fixed point, T a tracing point, and P a pencil, or marker. F, T, and P must always be in a straight line; FcT and FbP must always be similar triangles. In this instrument they are isosceles triangles. $abcT$ is always a parallelogram. The ratio of enlargement is $\frac{PF}{TF}$; to reduce, transpose P and T, that is, the marker will be placed at T and the tracer at *P* The diagram shows the fundamental principles, but, in practice, accuracy requires very fine workmanship and an expensive instrument. Very simple instruments are sometimes made by combining four sticks as shown in the sketch, using rivets at the joints, but the friction of the joints makes such simple instruments inaccurate. Fig. 152 shows a wooden pantograph in this simple form which may be bought for about $1.75. Fig. 153 shows another instrument called a *precision pantograph* which has the same essential geometrical principle, although the mechanical construction has a somewhat different outline, and the material, which is entirely of metal, has the very finest quality of workmanship. The cost varies with the length of the bars and is about $165.

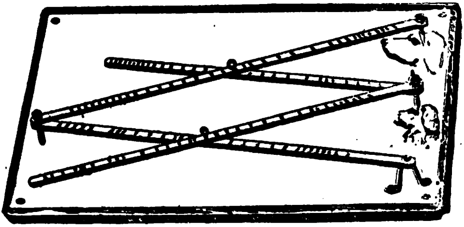


Fig. 152. Wooden Pantograph
Courtesy of Keuffel and Esser Company, New York City

Polar Planimeter. The polar planimeter is used to measure directly from any drawing the area of a given figure, no matter how irregular it may be. Although somewhat expensive (the one illustrated in Fig. 154 costing about $30), it is capable of great exactness and is a valuable mathematical invention which permits the recording on an index of the exact area of an irregular figure, simply by tracing the pointer around the perimeter of the figure. The polar planimeter, as its name implies, is worked around some fixed point, called the *pole*, which, in practice, means a needle point having a shoulder which is pricked into the paper exactly as is done with the needle point of a compass. The planimeter consists of two arms, the pole arm and the tracer arm. The pole arm carries the fixed point and is hinged to the tracer arm, which carries the tracing point and the measuring wheel with the mechanism for indicating the area. The hinge permits the tracing point to be moved away from the pole by an amount nearly equal to the combined length of the two arms, and if it is possible to fix the pole at some point from which every point in the perimeter of the figure may be reached by the tracer, then the area of the figure bmay be computed by a single setting of the instrument. If the figure is too large for this, then the area may be divided into two or more partial areas by lines which are preferably, although not neces-

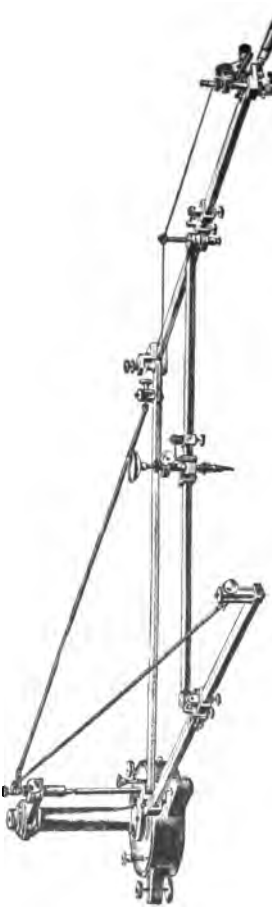


Fig. 153. Keuffel and Esser Type of Precision Pantograph

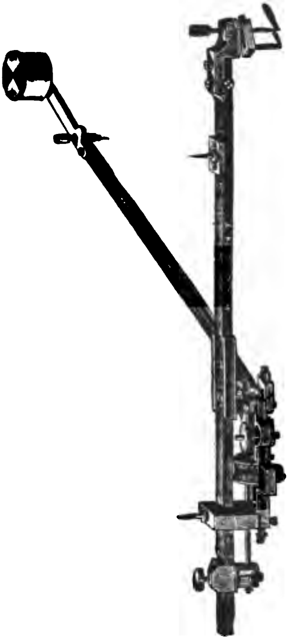


Fig. 154. Keuffel and Esser Type of Polar Planimeter

sarily, straight, and the area of each section may be computed separately. Perhaps the most common use of the polar planimeter is to compute the area of indicator cards, as described in our text on "Steam Engine Indicators". The planimeter has also another important use in the measurement of irregular ground areas, especially those which have been plotted by stadia methods rather than by the courses and distances of the boundary lines, and which cannot be computed by the common rule of a summation of products of latitudes and departures. It is thus possible to compute an area by surveying it rapidly by the stadia method, plotting it with desired accuracy at a suitable scale, and then measuring the area with a polar planimeter.

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